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Dynamic firm behavior under uncertainty

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DEDICATION

To my wife Marta

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ABBREVIATIONS, CONVENTIONS AND SYMBOLS

Abbreviations

CARA: constant absolute risk aversion

DARA: decreasing absolute risk aversion

FOC: first order condition

IARA: increasing absolute risk aversion

SOC: second order condition

USDA: United States Department of Agriculture

USDC: United States Department of Commerce

Conventions

Vectors are characterized by bold letters.

When there is no ambiguity, first and second derivatives are identified either by the symbols ' and ", respectively, or by subscripts.

Prices and costs are always denoted by small-case letters, and decision variables by capital letters. Whenever possible, parameters are designated by Greek letters.

The subscript t always refers to date t, while the subscript T stands for the terminal date. Superscripts o and m are used to characterize soybean oil and soybean meal, respectively.

Symbols

$c(\cdot)$ = nonmaterial cost function

$d(\cdot)$ = differential of $(\cdot)$

$\mathbf{d}_t$ = vector of relevant decision variables

e_t = vector of relevant predetermined and exogenous variables

$F(\cdot)$ = cumulative distribution function of the standard normal distribution

F_t = net short position for delivery of final product at date t+1 open at time t

f_t, $f_{t,t+k}$ = forward prices of final product at date t for delivery at times t+1 and t+k, respectively

F_t^s = net short position for delivery of material input at date t+1 open at time t

f_t^s, $f_{t;t+h}^s$ = forward prices of material input at date t for delivery at times t+1 and t+h, respectively

I_t, I_t^s = beginning inventories of final product and material input, respectively

$i(\cdot)$, $i^s(\cdot)$ = storage cost functions of final product and material input, respectively

P_t = sales of final product

p_t = price of final product

Q_t = production of final good

$q(\cdot)$ = production function

Q_t^s = use of material input

r_t = one plus one-period interest rate

S_t = purchases of material input

s_t = price of material input

$U_t(\cdot)$ = utility function

V_t = vector of nonmaterial inputs

v_t = vector of nonmaterial input prices

W_t = monetary wealth at end of trading date t

Φ = fixed input-output coefficient, $\Phi > 0$

η_t, η_t^s = Lagrangian multipliers corresponding to inventories of final product and material input, respectively

λ = Arrow-Pratt coefficient of absolute risk aversion, $\lambda > 0$

π_t = cash flow

$\mathbb{E}_t(\cdot)$ = expectation operator based on the information available at date t

$\pounds_t$ = Lagrangian function

Cov(x, y) = covariance between x and y

exp(x) = base of natural logarithms raised to the power x

ln(x) = natural logarithm of x

lag(x_t) = x_{t-1}

max($\cdot$) = maximum of ($\cdot$)

min($\cdot$) = minimum of ($\cdot$)

Var(x) = variance of x

CHAPTER I. INTRODUCTION

The theory of the competitive firm under conditions of uncertainty has been developed under the assumptions that the firm maximizes the expected utility of terminal profits, or wealth, and that the firm is *myopic.* A myopic firm can be defined as one whose planning horizon is the same as its decision horizon, which is equal to one period.[1] The work that has been done in this field is extensive, but the basic setting used for most studies is the model advanced by Sandmo (1971).

Imposing the myopic constraint greatly simplifies the analysis, but at the cost of severely restricting the firm's behavior. A myopic firm behaves as if it intends to exit the market immediately after finishing the current production cycle. A firm that plans to remain in business may respond to risk differently than an otherwise identical myopic firm. One reason for the differing responses is the commonly observed positive correlation between output and input prices. This correlation may result in current production serving as a hedge for subsequent input purchases, which in turn means that expectations about future production (which depends on future input purchases) may affect current production even if everything else remains unchanged. This effect of expectations about events occurring after the current production cycle cannot be analyzed with a myopic model, because this model implicitly assumes that future output is zero.

Comparatively few studies have relaxed the myopic restriction to allow the firm to be *forward-looking*, or *nonmyopic*. A forward-looking firm is one whose planning horizon

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[1]According to Merton (1982, p. 656), the planning horizon "is the maximum length of time for which the investor gives any weight in his utility function," and the decision horizon is "the length of time between which the investor makes successive decisions, and it is the minimum time between which he would take any action."

comprises at least two decision horizons. Where nonmyopic behavior has been studied, the analyses generally restrict utility to be intertemporally additive and prices to be independently distributed from one period to the next (Newbery and Stiglitz 1981, Hey 1987). These are strong assumptions: intertemporal additivity means perfect substitutability among single-period utilities, and price independence is not supported by empirical research. Studies on the forward-looking firm employing different assumptions include Zabel (1971), Anderson and Danthine (1983), Chavas (1988), and Chavas, Kristjanson, and Matlon (1991). Zabel postulated a constant absolute risk-averse (CARA) intertemporal utility function but constrained prices to be independent from period to period. Chavas used the mean-variance framework to analyze speculative storage, but the truncated shape of the storage function raises doubts about the validity of employing the mean-variance model in such a case.

The goal of this dissertation is to derive results highlighting the shortcomings of the myopic model vis-a-vis the forward-looking paradigm. We provide intuitive explanations for the results obtained and test some of the theoretical hypotheses advanced by applying them to the U.S. soybean-processing industry. The study proceeds as follows: first, we introduce the basic theoretical model by postulating and examining optimal decisions for a firm whose only activity is speculative storage. We subsequently modify the basic model to study a firm that produces and does not store. In Chapter III, we further develop each case to allow for forward trading. Then, in Chapter IV, we present the most general scenario in which the competitive firm produces and holds input and output inventories and also trades in input and output forward markets. Also in Chapter IV, we perform an empirical test of some of the theoretical hypotheses that emerge from the most general model using data from the U.S. soybean-processing industry. Finally, in Chapter V, we summarize the major results of the study.

Literature Review[2]

The seminal study in the theory of the firm under uncertainty was done by Sandmo (1971). The basic assumptions of his model are that the firm is myopic and that production is nonstochastic. His main finding was that the risk-averse firm will produce at a point at which the expected output price exceeds marginal cost, which means that production under price uncertainty will be lower compared with that under certainty, given the same expected price in both situations. Also, the risk-averse firm will produce a finite amount even if the marginal cost of production is constant, and a higher fixed cost will lead to lower production if the firm's utility function is decreasing absolute risk averse (DARA). Ishii (1977) showed that, in Sandmo's framework, a marginal increase in price uncertainty leads unambiguously to lower production if the firm's attitude is either DARA or CARA.

The hypotheses resulting from the works of Sandmo and Ishii led many to include uncertainty in the empirical estimation of supply curves and to try to quantify its impact. Examples include Just (1974); Lin (1977); Brorsen, Chavas, and Grant (1987); Anderson and Garcia (1989); and Chavas and Holt (1990). These studies generally resort to different kinds of ad hoc indexes of "riskiness" to build uncertainty into the models, some of which were compared in Traill (1978). Recently, Aradhyula and Holt (1989) employed the theoretically appealing GARCH time-series process to estimate expectations of means and variances of random variables in a model of the U.S. broiler market addressing both risk and rational expectations.

Danthine (1978); Holthausen (1979); and Feder, Just, and Schmitz (1980) further refined Sandmo's model by introducing a forward market for output. They proved that the competitive risk-averse firm that is able to trade forward separates production from hedging

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[2]This literature review is not exhaustive. The three results chapters (II, III, and IV) are self-contained papers that cite more specific references to previous works.

decisions if production is nonstochastic. Such a firm will produce as if output price were certain and equal to the forward price. Moreover, they showed that if the forward price is unbiased it is optimal to place a full hedge (i.e., to short hedge the entire output). Otherwise, it is optimal to short hedge more (less) than total production when the forward price is greater than (less than) the expected cash price.

Among the extensions to the pioneering work by Danthine; Holthausen; and Feder, Just, and Schmitz are analyses of production risk (Chavas and Pope 1982, Losq 1982, Honda 1983, Grant 1985), basis risk (Batlin 1983; Benninga, Eldor, and Zilcha 1984; Paroush and Wolf 1989), hedging costs (Chavas and Pope), hedging restrictions (Antonovitz and Roe 1986, Antonovitz and Nelson 1988), imperfect markets (Katz 1984), and option trading (Hanson 1988; Lapan, Moschini, and Hanson 1991; Hanson and Ladd 1991).

An interesting theoretical development is introduced in the paper by Brorsen et al. (1985), which focuses on marketing firms. Assuming that the marketing firm is in a competitive and/or contestable market, has either a DARA or CARA utility function, and has a Leontief nonstochastic production function, increasing output price risk should cause a higher expected marketing margin. The authors tested this hypothesis with data from the U.S. wheat-milling industry and found that it could not be rejected.

A shortcoming of the work by Brorsen et al. is that it does not take futures markets into account; hedging should be relevant because it allows the firm to reduce price risk as long as the basis is less volatile than the output price. Lence, Hayes, and Meyers (1992) adapted the models by Brorsen at al. and Benninga, Eldor, and Zilcha to address the problem of the marketing firm under uncertainty and in the presence of futures markets. Lence, Hayes, and Meyers tested their model with data from the U.S. soybean-processing industry and obtained encouraging results when compared to more traditional approaches. But Lence, Hayes, and Meyers were concerned only with processing because they used annual data, and they overlooked the fact that material input purchases and output sales may vary considerably from month to month within the same crop year.

There is also an important body of literature devoted to the analysis of input demand under uncertainty and in which it is assumed that the firm is myopic. Batra and Ullah (1974) pioneered this field, finding that changes in input prices may have a different impact under output price uncertainty than under certainty. Although some of their conclusions were faulty, as noted and corrected by Hartman (1975), their main results were correct: under output price uncertainty the production function must be well-behaved to obtain the standard certainty result that an increase in the price of an input unambiguously reduces its use. In addition, the impact of such a price rise on the demand for the other input cannot be predicted.

Stewart (1978) considered the case of a firm that combines an input with known price with another input whose price is random to produce a given quantity of output. He demonstrated that, if inputs are substitutes for each other, the risk-averse firm will use more of the input with known price and less of the risky input when compared to a risk-neutral firm. Therefore, the risk-averse firm will not produce at the input-usage ratio that minimizes expected cost. Moreover, if factor-price uncertainty increases, the risk-averse firm will use less of the risky input, leading to higher expected production costs. These results generalize to the situation in which there are multiple inputs and/or output price is random.

Perrakis (1980) made some amendments to Stewart's model. He showed that randomness in the prices of some inputs at the time when the level of other inputs must be chosen affects the proportion of inputs selected by the firm under any kind of risk attitude. In the most likely situation, even a risk-neutral firm will use a higher proportion of the riskless input compared to a situation in which the prices of the risky inputs are nonrandom and equal to their expectations.

Zabel (1971) pioneered the study of forward-looking behavior under risk aversion, allowing also for inventories of final product. Zabel postulated a CARA intertemporal utility function to characterize the preferences of the competitive firm. In the Zabel model, the firm produces a single product and in each period must decide how much to produce before the price is revealed and how much to sell after learning the price. The subjective density function of prices is identical and independent for each period, and the cost of production is assumed constant for the entire horizon. Zabel found that an increase in beginning inventories leads to a one-on-one increase in sales and to a decrease in production no larger than the increase in inventories. A bigger coefficient of absolute risk aversion translates unambiguously into lower production and higher sales; an increase in current price results in unchanged production and higher sales; and a decrease in the discount factor leads to higher production, but its effect on sales is ambiguous.

Chavas (1988) employed the mean-variance paradigm to study competitive speculation assuming a forward-looking decision maker. He demonstrated that the marginal risk premium may be either positive or negative depending on the expected change in future asset holding. An issue with Chavas's model, however, is that future asset holding is random and follows a truncated distribution. Because of this, assuming CARA utility and normally distributed prices does not lead to the mean-variance model, which may invalidate the conclusions.

Hey (1987) built a dynamic model of the competitive firm with a forward market for the final good. In that theoretical paper, Hey assumed a risk-averse firm with an additive intertemporal utility function. He also restricted output cash prices to be identically and independently distributed from date to date. The firm is allowed to hold inventories of final product and also to trade in a forward market for final product. With this setting, Hey proved that the firm separates production from hedging and showed that the firm produces as if the cash price were known and equal to the forward price. He also found that fully hedging total production is suboptimal when forward prices are unbiased. In addition to adopting the restrictive assumptions of additive utility and identically independent distributed cash prices, Hey's results depend crucially upon the sequential occurrence of the production, hedging, and sales decisions. In his model, the firm chooses optimal sales after having chosen production and hedging, rather than simultaneously.

A different approach to nonmyopic behavior in the presence of futures markets was undertaken by Anderson and Danthine (1983). They developed a model in which the firm revises its hedging decision between the dates at which its physical (i.e., cash) positions are open and closed. Anderson and Danthine found that separation between production and hedging holds, but that the full hedge is generally suboptimal if the futures/forward price is unbiased. However, they assumed a single production cycle, which can be very restrictive for some firms.

In summary, most of the work on the theory of the firm under uncertainty has assumed that the firm behaves myopically. Where this assumption has been relaxed, most models have either imposed severe constraints (i.e., additive intertemporal utility, independently distributed prices, or sequential production and selling decisions) or have not been directly concerned with the comparative analysis of forward-looking and myopic firms. That comparative analysis is the focus of this study.

CHAPTER II. COMPETITIVE FIRMS IN THE ABSENCE OF FORWARD MARKETS

Few studies have relaxed Sandmo's implicit assumption that the risk-averse firm plans to quit production at the end of the current period. There is at least one reason to expect this assumption to matter. Consider an industry for which input and output prices are positively correlated and firms remain in production for several production cycles. Here, a firm's end-of-period cash flow includes the costs required to initiate production in the subsequent period. The positive effect of high output prices may be offset by higher input prices. In addition, having output to sell, even if produced at a loss, can act as a hedge against input prices. Firms operating in this environment will be concerned about revenue and cost risks at several points in time and may choose to offset risk in one period against risk in another and will diversify risk across time.

In this chapter we present a model of a risk-averse expected-utility-maximizing firm that is concerned about revenue and cost risks in future production periods and we use it to derive propositions that add to the richness of Sandmo's model.

Existing nonmyopic models have generally restricted utility to be intertemporally additive and prices to be independently distributed across time (Newbery and Stiglitz 1981, Hey 1987). These are strong assumptions because intertemporal additivity implies perfect substitution among single-period utilities, and price independence is not supported by empirical research. Other work exists upon which we can build. Zabel (1971) uses a CARA intertemporal utility function but assumes intertemporal price independence. Chavas (1988) presents a forward-looking mean-variance model of speculative storage. However, using the mean-variance paradigm in this setting is hard to justify because the random storage function has a truncated distribution.

In the next section, we introduce a forward-looking firm whose only productive activity is speculative storage (or asset holding). For this case, the correlation between input and output prices is most obvious and leads to a straightforward analysis of the firm's behavior. We also show that Sandmo-type behavior is nested within the more general model by restricting the firm to be myopic. Then, we allow the firm to be involved in a more standard productive activity and derive some propositions. The results for this more general case are derived at the cost of some additional assumptions about the technology set.

A Speculative Storing Firm

Consider a competitive firm with a twice continuously differentiable von Neumann-Morgenstern utility function, and assume that utility is strictly concave in its argument terminal wealth [$U(W_T)$, $U'(W_T) > 0$, $U''(W_T) < 0$].[3] Terminal wealth is

$$(2.1) \quad W_T = r_{-1}\, r_0\, r_1 \cdots r_{T-1}\, W_{-1} + r_0\, r_1 \cdots r_{T-1}\, \pi_0 + r_1 \cdots r_{T-1}\, \pi_1 + \ldots + r_{T-1}\, \pi_{T-1} + \pi_T$$

where W_t denotes monetary wealth at end of trading date t, π_t is cash flow at time t, and r_t equals one plus the one-period interest rate prevailing at t. Interest rate need not be constant over time, but it is restricted to be nonrandom. At each trading date $0 \leq t < T$ the firm can borrow and lend unlimited amounts of money for one period at the prevailing interest rate.

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[3]As noted by Katz (1983), the proper argument of utility is wealth and not profits, although the latter approach has been widely (and incorrectly) used.

It will become clear later that input price randomness plays a key role in the forward-looking scenario, so that we want to account for it explicitly. But allowing for input price randomness would render the model intractable, as suggested by the related literature (Batra and Ullah 1974, Hartman 1975 and 1976, Ratti and Ullah 1976, Wright 1984, Stewart 1978, Perrakis 1980). The easiest way to tackle this problem is to postulate a *speculative* firm whose only activity is storing a certain product to profit from its resale, in which case the relevant cash flow at any date $t \leq T$ is represented by:

$$(2.2) \quad \pi_t = p_t P_t - i(I_t - P_t) \qquad \text{s.t.} \quad I_t = I_{t-1} - P_{t-1} \geq 0$$

where P_t represents product sales at date t, $i(\cdot)$ is a convex storage cost function such that $i'(\cdot) > 0$,[4] and I_t is beginning inventory at date t. Positive sales means that the firm sells from beginning stocks, whereas negative sales means that the firm buys to store and sell at a later date. Sales cannot exceed beginning inventory (i.e., $P_t \leq I_t$). The amount $(I_t - P_t)$ is the unsold beginning inventory at date t, which is carried over at nonrandom storage cost $i(I_t - P_t)$ to become beginning inventory at time t+1 (I_{t+1}). This kind of cash flow reduces the problem to its essentials and is generalized later.

Assume that at any moment t the firm chooses current product sales (P_t) to maximize expected utility of terminal wealth given available information (e_t). In addition, costless information becomes available between trading dates. Therefore, optimal sales level at current date $t = 0$ is obtained by solving the dynamic programming problem

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[4]For a risk-averse firm, $i''(\cdot) = 0$ yields a bounded solution. This is important because $i''(\cdot) \equiv 0$ is a quite common situation in the real world (for example, gold and common stock are most likely carried over at constant marginal storage cost). In contrast, for a risk-neutral firm we need $i''(\cdot) > 0$ for the solution to be bounded.

(2.3) $M_t(W_T, e_t) = \max_{P_t \leq I_t} £_t(W_T) | e_t$

where: $£_T(W_T) = U(W_T) + \eta_T (I_T - S_T)$

$$£_t(W_T) = \int_{p_{t+1}} M_{t+1}(W_T, e_{t+1})\, p_{t+1}(p_{t+1} | p_0, \ldots, p_t)\, dp_{t+1} + \eta_t (I_t - S_t), \quad 0 \leq t < T$$

$£_t$ is the Lagrangian function of the optimization problem, e_t is a vector containing all relevant predetermined and exogenous variables at date t, η_t is the Lagrangian multiplier, $p_{t+1}(p_{t+1} | p_0, \ldots, p_t)$ is the conditional density function of p_{t+1} given $(p_0, \ldots, p_t)$, and terminal wealth and cash flows are given by (2.1) and (2.2), respectively.

The first-order conditions (FOCs) corresponding to the terminal date T are

(2.4) $\dfrac{\partial £_T}{\partial P_T} = (p_T + i')\, M_T' - \eta_T = 0$

(2.5) $\dfrac{\partial £_T}{\partial \eta_T} = I_T - P_T = 0,\ \eta_T > 0,\ \eta_T \dfrac{\partial £_T}{\partial \eta_T} = 0$

The first term of the derivative of the Lagrangian function with respect to sales is positive, hence the Lagrangian multiplier (η_T) is also positive to satisfy (2.4). Because $\eta_T = 0$, $\dfrac{\partial £_T}{\partial \eta_T}$ must equal zero to avoid violating the Kuhn-Tucker condition (2.5), so that the optimal sales policy at the terminal date is to sell all beginning inventories (e.g., $P_T = I_T$). Therefore, optimal cash flow at the terminal date reduces to $\pi_T = p_T I_T$, and maximum attainable utility is

(2.6) $M_T(W_T, e_T) = U(r_{T-1} W_{T-1} + p_T I_T)$

For all dates prior to the terminal date ($0 \le t < T$), the FOCs are[5]

$$(2.7)\quad \frac{\partial £_t}{\partial P_t} = r_{t+1} \cdots r_{T-1} \, [r_t \, (p_t + i') \, M_t' - \mathbb{E}_t(p_{t+1} \, M_{t+1}')] - \eta_t = 0$$

$$(2.8)\quad \frac{\partial £_t}{\partial \eta_t} = I_t - P_t \ge 0, \; \eta_t \ge 0, \; \eta_t \, \frac{\partial £_t}{\partial \eta_t} = 0$$

where $\mathbb{E}_t(\cdot)$ denotes the expectation operator based upon the information available at t. The solution to FOCs (2.7) and (2.8) is a unique absolute constrained maximum because the objective function is strictly concave, and the constraint set is convex.[6] Expression (2.6) together with FOCs (2.7) and (2.8) gives us the framework needed to analyze the behavior of the nonmyopic risk-averse firm.

A myopic firm is one whose planning and decision horizons are identical, whereas a forward-looking firm is defined by a planning horizon that consists of at least two decision horizons. The definition of myopic firm leaves two possibilities: the firm is either at terminal date T, or at time T-1. But at T the firm faces no risk, and therefore by myopic behavior under uncertainty we mean the behavior of the firm at date T-1. Similarly, a forward-looking (or nonmyopic) firm is one optimizing at any date before T-1.

Because we will compare the risk-averse firm with an otherwise identical risk-neutral firm, we need to know the optimal behavior of the latter. It is straightforward to show that the risk-neutral FOCs for any date preceding the terminal time T are[7]

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[5]See Appendix A for the derivation of (2.7).

[6]We will assume for the remainder of the analysis that the solution to (2.3) exists. The conditions for this are given in Bertsekas (1976, p. 375).

[7]Expression (2.9) is derived in Appendix A.

(2.9) $$\frac{\partial \pounds_t}{\partial P_t} = r_{t+1} \cdots r_{T-1} [r_t (p_t + i') - E_t(p_{t+1})] - \eta_t = 0$$

(2.10) $$\frac{\partial \pounds_t}{\partial \eta_t} = I_t - P_t \geq 0, \eta_t \geq 0, \eta_t \frac{\partial \pounds_t}{\partial \eta_t} = 0$$

and that optimal sales policy at T is given by $P_T = I_T$ (see Appendix A). It follows immediately from FOCs (2.9) and (2.10) that in the risk-neutral context optimal myopic and forward-looking sales are identical, so that we will not distinguish between myopic and nonmyopic risk neutrality.

We can obtain comparative statics corresponding to the risk-averse firm by total differentiation of FOCs (2.7) and (2.8). The myopic and forward-looking responses of sales and storage to current price, beginning inventories, the degree of absolute risk aversion, the interest rate, and parameters of the distribution of next-date price are summarized in Propositions 2.1 and 2.2, respectively. Note that we use the acronym IARA to denote increasing absolute risk aversion.

<u>Proposition 2.1: Myopic storage and sales behavior</u> For any positive amount stored, a myopic risk-averse firm behaves as follows:

a) Sales:

$$\frac{\partial P_{0=T-1}}{\partial p_0} \begin{cases} > 0 \text{ if CARA; or DARA and } P_0 \leq 0\text{; or IARA and } P_0 \geq 0 \\ \gtrless 0 \text{ if DARA and } P_0 > 0\text{; or IARA and } P_0 < 0 \end{cases}$$

$$\frac{\partial P_{0=T-1}}{\partial r_0} \begin{cases} > 0 \text{ if CARA; or DARA and } p_0 P_0 \leq i\text{; or IARA and } p_0 P_0 \geq i \\ \gtrless 0 \text{ if DARA and } p_0 P_0 > i\text{; or IARA and } p_0 P_0 < i \end{cases}$$

$$\frac{\partial P_{0=T-1}}{\partial I_0} \begin{cases} < 1 \text{ if DARA} \\ = 1 \text{ if CARA} \\ > 1 \text{ if IARA} \end{cases}$$

$$\left.\frac{\partial P_{0=T-1}}{\partial \mu_{0;1}}\right|_{\sigma=1} \begin{cases} < 0 \text{ if DARA or CARA} \\ \gtrless 0 \text{ if IARA} \end{cases}$$

$$\left.\frac{\partial P_{0=T-1}}{\partial \sigma_{0;1}}\right|_{\sigma=1} \begin{cases} > 0 \text{ if DARA or CARA} \\ \gtrless 0 \text{ if IARA} \end{cases}$$

$$\frac{\partial P_{0=T-1}}{\partial \lambda} > 0$$

b) Storage:

$$\frac{\partial I_{1=T}}{\partial p_0} \begin{cases} < 0 \text{ if CARA; or DARA and } P_0 \leq 0\text{; or IARA and } P_0 \geq 0 \\ \gtrless 0 \text{ if DARA and } P_0 > 0\text{; or IARA and } P_0 < 0 \end{cases}$$

$$\frac{\partial I_{1=T}}{\partial r_0} \begin{cases} < 0 \text{ if CARA; or DARA and } p_0\, P_0 \leq i\text{; or IARA and } p_0\, P_0 \geq i \\ \gtrless 0 \text{ if DARA and } p_0\, P_0 > i\text{; or IARA and } p_0\, P_0 < i \end{cases}$$

$$\frac{\partial I_{1=T}}{\partial I_0} \begin{cases} > 0 \text{ if DARA} \\ = 0 \text{ if CARA} \\ < 0 \text{ if IARA} \end{cases}$$

$$\left.\frac{\partial I_{1=T}}{\partial \mu_{0;1}}\right|_{\sigma=1} \begin{cases} > 0 \text{ if DARA or CARA} \\ \gtrless 0 \text{ if IARA} \end{cases}$$

$$\left.\frac{\partial I_{1=T}}{\partial \sigma_{0;1}}\right|_{\sigma=1} \begin{cases} < 0 \text{ if DARA or CARA} \\ \gtrless 0 \text{ if IARA} \end{cases}$$

$$\frac{\partial I_{1=T}}{\partial \lambda} < 0$$

where: $p_1 = \sigma_{0,1}\, p_1 + (1 - \sigma_{0,1})\, \mu_{0,1}$, $\sigma_{0,1}$ = constant, $\mu_{0,1} = \mathbb{E}_0(p_1)$

λ = Arrow-Pratt coefficient of absolute risk aversion ($\lambda = - M_1''/M_1'$)

Proof: See Appendix A.

Proposition 2.2: Forward-looking storage and sales behavior For any positive amount stored, the sales and storage responses of a forward-looking risk-averse firm to changes in current price, interest rate, beginning inventories, expected next-date price, mean-preserving price spread, and degree of absolute risk aversion are ambiguous in general. But if the firm is CARA, behavior is as follows:

a) Sales:

$$\frac{\partial P_{0<T-1}}{\partial p_0} > 0, \quad \frac{\partial P_{0<T-1}}{\partial r_0} > 0, \quad \frac{\partial P_{0<T-1}}{\partial I_0} = 1$$

b) Storage:

$$\frac{\partial I_{1<T}}{\partial p_0} < 0, \quad \frac{\partial I_{1<T}}{\partial r_0} < 0, \quad \frac{\partial I_{1<T}}{\partial I_0} = 0$$

Proof: See Appendix A.

By noting that storage (I_1) is the "productive" activity of the speculative firm, we see that Proposition 2.1 is a restatement of the findings of the standard literature on the firm under uncertainty for the case of speculative storage. If the myopic firm is DARA or CARA, storage increases with higher expected price or lower Rothschild-Stiglitz mean-preserving price spread. Also, myopic storage is negatively related to the firm's degree of absolute risk aversion.

From Proposition 2.1, beginning stocks have a positive (negative) effect on storage if the firm is DARA (IARA). This result is to be expected: a *ceteris paribus* increase in beginning stocks makes the firm wealthier and therefore less absolute risk averse if DARA. But we know that storage is negatively associated to the degree of absolute risk aversion, so that storage grows when beginning stocks increase for a DARA firm.

The ambiguous response to current price in Proposition 2.1 may seem counterintuitive. One would expect current price to affect storage negatively because current price may be considered an input price for storage. But a current price change also causes a wealth change, and consequently a change in the degree of absolute risk aversion unless the firm is CARA. For the CARA firm the degree of absolute risk aversion does not depend on current price, and the storage response to current price is unambiguously negative. Also, the DARA firm that buys good to store (i.e., $P_0 \leq 0$) reduces storage as current price increases. Otherwise, DARA storage bears an ambiguous relationship with current price. A similar explanation can be given to the counterintuitive result regarding the interest rate in Proposition 2.1.

When the firm is CARA the degree of absolute risk aversion is unaffected by a change in a particular exogenous variable. Therefore, the CARA firm's response does not include the indirect effect due to the exogenous variable impact on the degree of absolute risk aversion. For non-CARA firms this indirect effect may be of opposite direction and sufficiently large so as to outweigh the exogenous variable direct effect, which is the reason for the ambiguities that arise from DARA or IARA attitudes in Proposition 2.1. Because of these ambiguities, non-CARA forward-looking behavior cannot be characterized without imposing more restrictions (see Proposition 2.2).

If we constrain the nonmyopic firm to be CARA, we obtain unambiguous responses to current price, interest rate, and beginning inventory. Moreover, these responses are qualitatively the same as for the myopic CARA firm. But the effect of next-date expected price, next-date Rothschild-Stiglitz mean-preserving price spread, and the coefficient of absolute risk aversion on forward-looking sales and storage are ambiguous even for CARA. This result is counterintuitive and stands in contrast with what was found for the myopic case. This finding merits a more careful analysis because it challenges some of the main conclusions of the standard theory of the competitive firm under uncertainty. But we can show that it is a plausible characterization of real-world firm behavior. To explain this result, it is helpful to rewrite FOC (2.7) in terms of the covariance.[8] If the firm stores something at the present date (e.g., $I_1 = I_0 - P_0 > 0$), the Lagrangian multiplier must equal zero ($\eta_0 = 0$) and we can express (2.7) as

$$(2.11)\quad \mathbb{E}_0(p_1) + \frac{\mathrm{Cov}(p_1;\, M_1')}{M_0'} = r_0\,[p_0 + i'(I_1)]$$

On the other hand, if the firm stores nothing ($I_1 = I_0 - P_0 = 0$) the Lagrangian multiplier is positive ($\eta_0 \geq 0$), and instead of (2.11) we have

$$(2.12)\quad \mathbb{E}_0(p_1) + \frac{\mathrm{Cov}(p_1;\, M_1')}{M_0'} \leq r_0\,[p_0 + i'(0)]$$

But M_0' is positive everywhere, and we can infer the sign of $\mathrm{Cov}(p_1, M_1')$ in expressions (2.11) and (2.12) by examining the response of M_1' to changes in p_1, i.e.,

$$(2.13)\quad \frac{\partial M_1'}{\partial p_1} = r_1 \ldots r_{T-1}\, I_1\, M_1'' - r_1 \ldots r_{T-1}\, I_2\, M_1'' + \frac{\partial M_1'}{\partial P_1}\,\frac{\partial P_1}{\partial p_1}$$

$$+ \max_{P_1 \leq I_1}\Big[\int_{p_2} M_2'\, \frac{\partial \rho_2(p_2 \mid p_0, p_1)}{\partial p_1}\, dp_2\Big] + \ldots + \max_{P_1 \leq I_1}\Big\{\int_{p_2} \max_{P_2 \leq I_2}\Big[\int_{p_3} \ldots$$

$$\int_{p_{T-1}} \max_{P_{T-1} \leq I_{T-1}}\Big(\int_{p_T} M_T'\, \frac{\partial \rho_T(p_T \mid p_0, \ldots, p_{T-1})}{\partial p_1}\, dp_T\Big)$$

$$\rho_{T-1}(p_{T-1} \mid p_0, \ldots, p_{T-2})\, dp_{T-1}\Big] \ldots \rho_2(p_2 \mid p_0, p_1)\, dp_2\Big\}$$

---

[8]Recall that for any pair of random variables x and y: $\mathbb{E}(x\, y) = \mathbb{E}(x)\, \mathbb{E}(y) + \mathrm{Cov}(x, y)$.

The term ($r_1 \ldots r_{T-1} I_1 M_1''$) reflects the impact of current storage, and is nonpositive. The term ($- r_1 \ldots r_{T-1} I_2 M_1''$) is due to the effect of next-date storage, and is nonnegative. The third term in the right-hand side of (2.13) captures the impact of changes in absolute risk aversion, and vanishes for a CARA decision maker. Finally, the terms $\max_{P_1 \leq I_1}(\cdot)$ represent the effect of next-date price attributable to its relationship with posterior prices.

Expressions (2.9) through (2.13) give us the elements to derive our second set of results, which are summarized in Propositions 2.3 and 2.4, and their respective Corollaries.

Proposition 2.3: Myopic reservation price for storage The reservation price above which a myopic risk-averse firm does not store is equal to the risk-neutral reservation price. A myopic risk-averse firm will store at a level where discounted expected next-date price is higher than current price plus marginal storage cost.

Proof. The risk-neutral reservation price is $p^{m}_{0<T} = E_0(p_1)/r_0 - i'(0)$; the proof is trivial from FOCs (2.9) and (2.10).

For a myopic firm $I_2 = I_{T+1} = 0$, and the right-hand side of (2.13) reduces to $I_T M_T''$. Therefore,

$$(2.14) \quad \mathrm{Cov}(p_T, M_T') \begin{cases} < 0 \text{ if } I_T > 0 \\ = 0 \text{ if } I_T = 0 \end{cases}$$

because p_T is monotonically increasing in p_T, and M_T' is monotonically nonincreasing in p_T.[9] Applying expression (2.14) to (2.11) and (2.12) we get

[9]This result is obtained by employing Theorem 43 in Hardy, Littlewood, and Pólya (1967).

$$(2.15)\ \mathbb{E}_{T-1}(p_T) \begin{cases} > r_{T-1}\ [p_{T-1} + i'(I_T)] \text{ if } I_T > 0 \\ \leq r_{T-1}\ [p_{T-1} + i'(I_T)] \text{ if } I_T = 0 \end{cases}$$

Hence, the myopic risk-averse reservation price is $p^{ra}_{0=T-1} = \mathbb{E}_0(p_1)/r_0 - i'(0) = p^{rn}_{0<T}$. Q.E.D.

Corollary to Proposition 2.3 The myopic risk-averse firm stores less than the risk-neutral firm.

Proposition 2.4: Forward-looking reservation price for storage (1) The reservation price above which a forward-looking risk-averse firm does not store is generally different from the risk-neutral or the myopic risk-averse reservation price. Moreover, this firm does not necessarily store an amount at which discounted expected next-date price is higher than current price plus marginal storage cost.

(2) If the firm is CARA and (a) next-date price is independently distributed from all posterior prices, or (b) the decision maker is sufficiently absolute risk-averse, or (c) price follows a stationary autoregressive process and the decision maker is sufficiently forward-looking, then the forward-looking reservation price is higher than the risk-neutral or the myopic risk-averse reservation price.

Proof. For a nonmyopic firm the terms $\max_{P_1 \leq I_1}(\cdot)$ in expression (2.13) have ambiguous signs, even if $I_1 = 0$. Therefore, $\text{Cov}(p_1, M_1') \gtrless 0$, and $\mathbb{E}_0(p_1) \gtrless r_0\ [p_0 + i'(I_1)]$ for $I_1 \geq 0$. In particular, the forward-looking risk-averse reservation price $p^{ra}_{0<T-1}$ is such that $p^{ra}_{0<T-1} \gtrless \mathbb{E}_0(p_1)/r_0 - i'(0) = p^{ra}_{0=T-1} = p^{rn}_{0<T}$.

By FOC, $\partial M_1'/\partial P_1 = \partial M_1/\partial P_1 = 0$ if the firm is CARA.

The result under assumption (a) follows immediately because expression (2.13) simplifies to $(- r_1 \ldots r_{T-1}\ I_2\ M_1'') \geq 0$ if next-date price is independent from all posterior prices. Therefore, $\mathrm{Cov}(p_1, M_1') > 0$ unless $E_0(I_2) = 0$, and the forward-looking CARA reservation price $p^{CARA}_{0<T-1}$ is such that $p^{CARA}_{0<T-1} > E_0(p_1)/r_0 - i'(0) = p^{ra}_{0=T-1} = p^{rn}_{0<T}$.

To show the finding under hypothesis (b), let $\lambda = -M_t''/M_t'$ and re-express (2.13) to get

$$(2.16)\quad \frac{\partial M_1'}{\partial p_1} = -r_1 \ldots r_{T-1} I_2 M_1'' - \max_{p_1 \le I_1}\left[\int_{p_2} \frac{M_2''}{\lambda} \frac{\partial p_2(p_2 | p_0, p_1)}{\partial p_1} dp_2\right]$$

$$- \ldots - \max_{p_1 \le I_1}\left\{\int_{p_2} \max_{p_2 \le I_2}\left[\int_{p_3} \ldots\right.\right.$$

$$\int_{p_{T-1}} \max_{p_{T-1} \le I_{T-1}}\left(\int_{p_T} \frac{M_T''}{\lambda} \frac{\partial p_T(p_T | p_0, \ldots, p_{T-1})}{\partial p_1} dp_T\right)$$

$$\left.\left. p_{T-1}(p_{T-1} | p_0, \ldots, p_{T-2})\, dp_{T-1}\right] \ldots p_2(p_2 | p_0, p_1)\, dp_2\right\}$$

Unless $E_0(I_2) = 0$, we can obtain $\mathrm{Cov}(p_1, M_1') > 0$ by setting λ large enough, because

$$(2.17)\quad \lim_{\lambda \to \infty} \frac{\partial M_1'}{\partial p_1} = -r_1 \ldots r_{T-1} I_2 M_1'' \ge 0$$

The result under assumption (c) can be proven similarly. The relationship between next-date price and prices close to the terminal date tends to vanish as the planning horizon lengthens, i.e., $\partial p_t/\partial p_1 \equiv 0$ for $(t-1)$ sufficiently large. Therefore, for sufficiently forward-looking behavior the first term in (2.16) outweighs the terms $\max_{p_1 \le I_1}(\cdot)$, thus yielding $\mathrm{Cov}(p_1, M_1') > 0$. Q.E.D.

<u>Corollary to Proposition 2.4</u> If conditions (a), (b), or (c) in Proposition 2.4 are met, there exists a range of current prices over which the forward-looking CARA firm stores more than the risk-neutral one.

Proposition 2.3 and its Corollary extend well known results from the theory of the firm under uncertainty to the myopic speculative storage scenario. Proposition 2.4 and its Corollary contain some of the key findings of this chapter and provide the intuition for the seemingly paradoxical results of Proposition 2.2. Comparison of Propositions 2.3 and 2.4 (and their respective Corollaries) reveals that relaxing the myopic assumption yields nontrivial differences in speculative storing behavior.

It is important to stress that in the forward-looking scenario we cannot use normally distributed prices to justify mean-variance analysis because terminal beginning inventory is random but it is not normally distributed. From Proposition 2.3, it follows that $I_T = 0$ when current price is above the myopic reservation price [i.e., when $p_{T-1} > \mathbb{E}_{T-1}(p_T)/r_{T-1} - i'(0)$]. This creates a truncation point in the density function of terminal wealth.

In Figures 2.1 and 2.2 we illustrate the most important findings reported in Propositions 2.1 through 2.4. Figure 2.1 is drawn in storage-current price space, whereas Figure 2.2 is done in sales-current price space. In each Figure we depict the curves "myopic CARA," "forward-looking CARA," and "risk-neutral" to represent the hypothetical behavior of three firms assumed identical except for their planning horizons and risk attitudes. The slope of the storage curves for the CARA firms is negative (see Propositions 2.1 and 2.2). Also, the CARA storage curves are steeper than the risk-neutral one, as inferred from the equations giving the storage response to current price (see Appendix A), i.e.,

$$(2.18)\quad \left|\frac{\partial I_{0<T}^{CARA}}{\partial p_0}\right| = \frac{1}{i'' - r_0 \cdots r_{T-1}\,\mathbb{E}_0\{[(p_0 + i') - p_1/r_0]^2\, M_1''\}/M_0'} < \frac{1}{i''} = \left|\frac{\partial I_{0<T}^{rn}}{\partial p_0}\right|$$

for $I_{0<T}^{CARA} = I_{0<T}^{rn}$

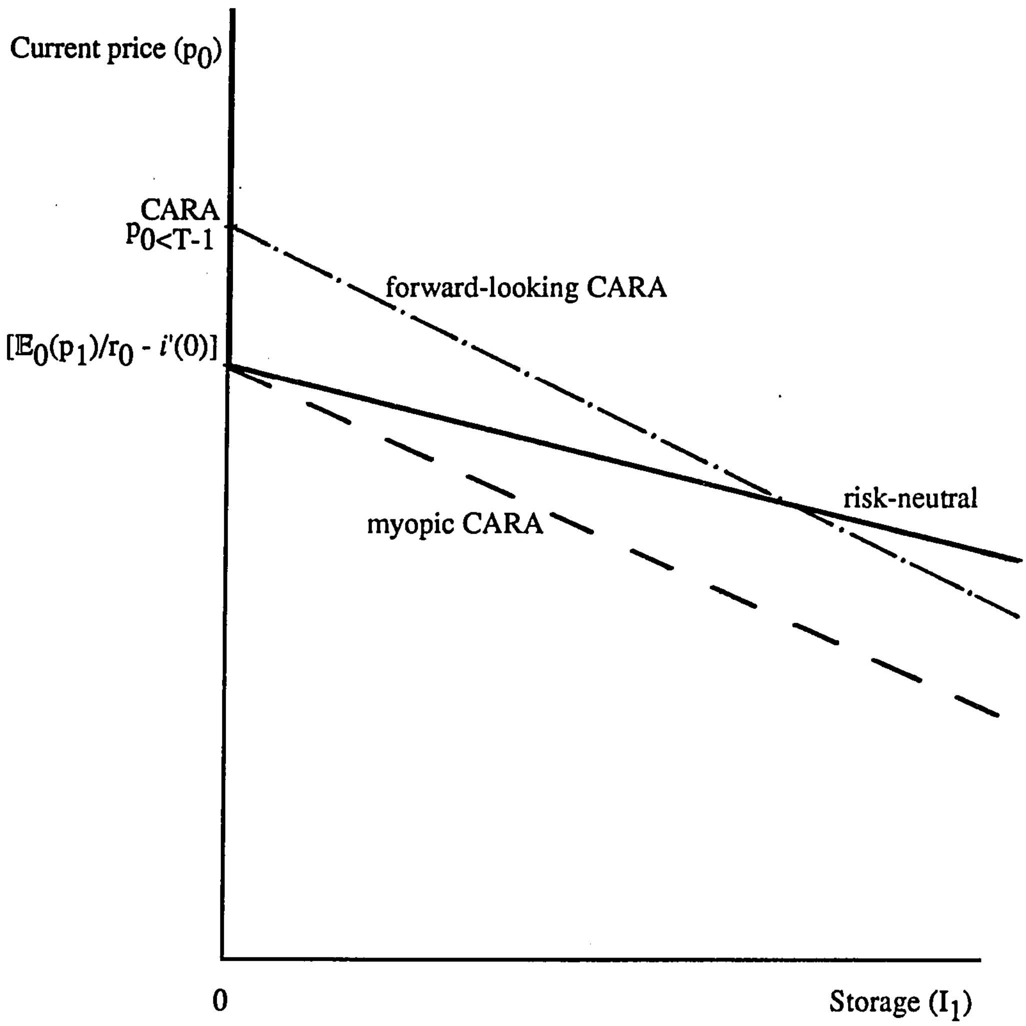

Figure 2.1. Storage behavior of risk-neutral, myopic CARA, and nonmyopic CARA firms

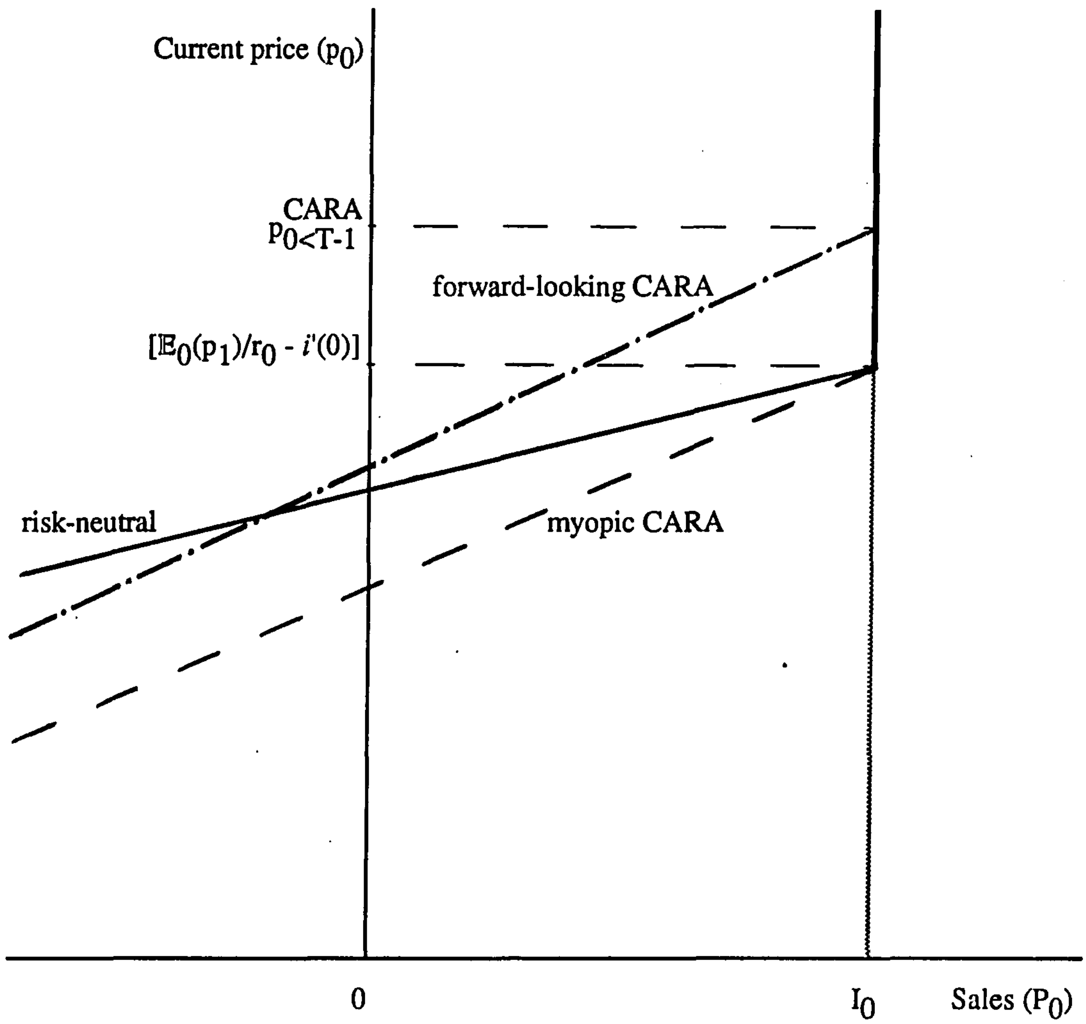

Figure 2.2. Sales behavior of risk-neutral, myopic CARA, and nonmyopic CARA firms

where |x| represents the absolute value of x, and the superscripts "CARA" and "rn" stand for CARA and risk-neutral firms, respectively.

As stated in Proposition 2.3, the risk-neutral and myopic CARA reservation prices are identical in Figures 2.1 and 2.2. Also, risk-neutral storage is always above myopic storage. Figures 2.1 and 2.2 represent the case in which conditions (a), (b), or (c) of Proposition 2.4 are met, so that the forward-looking CARA reservation price is above the risk-neutral one. Because storage curves are negatively sloped, the nonmyopic CARA firm stores more than a risk-neutral one when current price is between the risk-neutral and the forward-looking reservation prices (i.e., $I_{0<T-1}^{CARA} > I_{0<T}^{rn} = 0$ if $p_{0<T-1}^{CARA} > p_0 > p_{0<T}^{rn}$). Moreover, if storage cost is a strictly convex function (as depicted), forward-looking CARA storage will also exceed risk-neutral storage for some range of current prices below the risk-neutral reservation price (i.e., $I_{0<T-1}^{CARA} > I_{0<T}^{rn} > 0$ for some $p_0 < p_{0<T}^{rn}$).

When current price is between the forward-looking CARA and the risk-neutral reservation prices, we observe a decrease in nonmyopic CARA storage as we reduce the coefficient of absolute risk aversion from some positive value to zero (i.e., as firms become risk-neutral). This is the reason why forward-looking CARA storage may increase with the degree of absolute risk aversion. We can apply a similar reasoning to explain the ambiguous effect of next-date expected price and next-date Rothschild-Stiglitz mean-preserving spread on forward-looking CARA storage.

From the proof of Proposition 2.4, it is clear that if current storage is sufficiently high we will have $\mathrm{Cov}(p_1, M_1') < 0$, because the first term in the right-hand side of (2.13) will outweigh the other terms. Therefore, for sufficiently high current storage we will have risk-neutral storage exceeding forward-looking CARA storage. Also, because of inequality (2.18), the forward-looking and risk-neutral curves will intersect at a unique point. These observations are depicted in Figure 2.1.

We can readily explain the *apparent* irrationality of a nonmyopic CARA firm holding inventories where current price is above discounted expected next-date price minus storage cost. Let t_0, t_1, and t_2 be three arbitrary successive *calendar* times, and write the terminal cash flow of dates t_0 and t_1 in the following way:

$$(2.19)\quad r_{t_0}\,\pi_{t_0} + \pi_{t_1} = r_{t_0}\,[p_{t_0}\,P_{t_0} - i(I_{t_0} - P_{t_0})] + [p_{t_1}\,P_{t_1} - i(I_{t_1} - P_{t_1})]$$

$$= r_{t_0}\,[p_{t_0}\,I_{t_0} - p_{t_0}\,I_{t_1} - i(I_{t_1})] + [p_{t_1}\,I_{t_1} - p_{t_1}\,I_{t_2} - i(I_{t_2})]$$

When the myopic firm is at time t_0, its planning horizon ends at next date t_1, so that terminal date for the myopic firm standing at t_0 is $T = t_1$. If it behaves optimally, the myopic firm at date $T\text{-}1 = t_0$ will plan to sell its entire current storage at date $T = t_1$. Therefore, at time t_0 the myopic firm cares only about revenue risk at t_1 (i.e., $p_{t_1}\,I_{t_1}$). In contrast, the forward-looking firm's planning horizon ends after next date, so that at time t_0 its terminal date T is greater than t_1. Because of this, the forward-looking firm generally expects to store something at t_1 [i.e., $\mathbb{E}_{t_0}(I_{t_2}) > 0$], in which case it faces cost risk [i.e., $p_{t_1}\,I_{t_2} + i(I_{t_2})$] in addition to revenue risk from its activities at t_1. But revenue and input cost risks are related to each other and to current storage. In particular, current storage increases revenue risk but reduces input cost risk. This means that the forward-looking firm may derive utility from holding some inventory even when the one-period expected return from storage is negative, because storing reduces its cost risk. In a sense, the forward-looking firm diversifies assets intertemporally.

Our results are compatible with the findings of the standard theory of the firm under uncertainty because the standard results apply when the forward-looking firm stores a sufficiently large amount. But our model explains real-world facts that are incompatible with the standard model of the firm under uncertainty. For example, firms practice sequential marketing (Hanson, 1988, p. 6), hold output and/or input reserves, and spread transactions over time to reduce risk (Robison and Barry, 1987, p. 65).

The results in this section are quite general in the sense that they apply not only to firms speculating with commodity storage, but also to speculative holders of stocks, bonds, and other non-transformable assets.

To illustrate the preceding findings, consider the following example. Assume for simplicity that the storage cost function is quadratic and has the form $i(I_t) = \Theta I_t^2$, and that prices are independently normally distributed:

$$(2.20)\quad p_t \sim \text{n.i.d.}\ (\mu_t, \sigma_t^2)$$

Under these conditions, optimal storage levels for a risk-neutral, a myopic CARA, and a forward-looking CARA firm standing at date T-2 are respectively respectively obtained from:[10,11]

$$(2.21)\quad I_{1 \leq T} = \begin{cases} (\mu_T - r_{T-1}\, p_{T-1})/(2\, r_{T-1}\, \Theta) \text{ if } \mu_T > r_{T-1}\, p_{T-1} \\ 0 \text{ if } \mu_T \leq r_{T-1}\, p_{T-1} \end{cases}$$

$$(2.22)\quad I_{1=T} = \begin{cases} (\mu_T - r_{T-1}\, p_{T-1})/(2\, r_{T-1}\, \Theta + \lambda\, \sigma_T^2) \text{ if } \mu_T > r_{T-1}\, p_{T-1} \\ 0 \text{ if } \mu_T \leq r_{T-1}\, p_{T-1} \end{cases}$$

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[10]See Appendix A for the derivations of expressions (2.22) and (2.23). Expression (2.21) can be obtained from (2.22) by setting $\lambda = 0$.

[11]As we already know, $I_{T+1} = 0$ for any rational firm independently of price distributions.

(2.23) $K_0 \le 0,\ I_{1=T-1} \ge 0,\ I_{1=T-1}\, K_0 = 0$

$$\text{where: } K_0 = \frac{1}{\sqrt{2\pi}} \exp[-\tfrac{1}{2}(\mu_T/r_{T-1})^2]\,[1 - (1 + \sigma_{T-1}^{\;2} K_2)^{-1/2} \exp(-\tfrac{1}{2}\,\sigma_{T-1}^{\;2} K_2\, z_1^{\,2})]$$

$$+ (1 + \sigma_{T-1}^{\;2} K_2)^{-1/2} \exp(-\tfrac{1}{2}\,\sigma_{T-1}^{\;2} K_2\, z_1^{\,2})$$

$$[\mu_T/r_{T-1} - (1 + \sigma_{T-1}^{\;2} K_2)^{-1/2}\, \sigma_{T-1}\, z_1 - r_{T-2}\,(p_{T-2} + 2\,\Theta\, I_{1=T-1})]\, F(z_1)$$

$$+ [\mu_T/r_{T-1} - \sigma_{T-1}\, z_0 - r_{T-2}\,(p_{T-2} + 2\,\Theta\, I_{1=T-1})]\, F(-z_0)$$

$$z_0 = -(\mu_{T-1} - \mu_T/r_{T-1} - \lambda\, r_{T-1}\, \sigma_{T-1}^{\;2}\, I_{1=T-1})/\sigma_{T-1}$$

$$z_1 = (1 + \sigma_{T-1}^{\;2} K_2)^{-1/2}\, z_0$$

$$K_2 = r_{T-1}^{\;2}\, \lambda/(2\, r_{T-1}\, \Theta + \lambda\, \sigma_T^{\,2})$$

$F(\cdot)$ = cumulative distribution function of the standard normal distribution

We report some a specific numerical solution in Table 2.1. This table gives an example of a forward-looking CARA firm that stores more than a risk-neutral one.

A Model of Production without Storage

The main results discussed in the preceding section were obtained by assuming the cash flow presented in (2.2), and are due to the contemporaneous relationship between revenue and

Table 2.1. Example of a forward-looking CARA firm that stores more than a risk-neutral firm

	CARA firm[a]	Risk-neutral firm[b]
$I_{0=T-1}$	82 units	238 units
$I_{0=T-2}$	547 units	238 units

[a]Values of parameters and exogenous variables are $1/\lambda = 50000$ \$, $r_{T-1} = r_{T-2} = 1.05$, $\mu_T = 110$ \$/unit, $\mu_{T-1} = 100$ \$/unit, $p_{T-2} = 95$ \$/unit, $\sigma_T = 10$ \$/unit, $\sigma_{T-1} = 11$ \$/unit, and $\Theta = 0.0005$ \$/unit^2.

[b]Values of parameters and exogenous variables are $\lambda = 0$, $r_{T-1} = r_{T-2} = 1.05$, $\mu_T = 110$ \$/unit, $\mu_{T-1} = 100$ \$/unit, $p_{T-2} = 95$ \$/unit, $\sigma_T = 10$ \$/unit, $\sigma_{T-1} = 11$ \$/unit, and $\Theta = 0.0005$ \$/unit^2.

input cost at each date. In this section we will show that similar conclusions apply to firms characterized by less restrictive cash flows. The complications that arise from allowing for random input prices in a nonmyopic context are due to the possibilities of stochastic production and/or input substitution. Hence, we can apply our basic model to other types of cash flows by constraining the production function to be nonstochastic and such that inputs with random prices cannot be substituted.

It is straightforward to extend the analysis performed in the previous section to competitive firms whose short-run production function can be represented by a Leontief production function such as

$$(2.24)\quad Q_t = \min[Q_t^S/\Phi,\ q(V_t)]$$

where Q_t denotes production of final good at date t, $Q_t \geq 0$, Q_t^S represents material input use, Φ is a fixed input-output coefficient ($\Phi > 0$), V_t is a vector of nonmaterial inputs, and $q(\cdot)$ is a concave production function. Output Q_t becomes available only at date t+1; in other words, the firm starts production at time t and finishes output at date t+1.

According to (2.24), adding Φ units of material input increases production by one unit over the range in which the vector of nonmaterial inputs does not constrain production. If enough units of material input are added the set of nonmaterial inputs eventually becomes binding and production cannot increase. The fact that there is no substitutability between material input and $q(\cdot)$ does not mean that substitution among the nonmaterial inputs in vector V_t is prevented. For example, it may be feasible to substitute capital for labor in wheat milling, even though substitability of wheat for either of these two inputs combined or alone is negligible for all practical purposes. Note also that material input becomes nonbinding as Φ

tends to zero, resulting in a standard production function $q(\cdot)$. In other words, the standard production function is nested in (2.24).

For our purposes it is essential that the Leontief function (2.24) is nonstochastic and that there is no substitution between material and nonmaterial inputs. This allows us to examine the situation where material input price is random without the complications due to input substitution or stochastic output. Storage, transportation, refining and/or purifying of raw materials (e.g., oil, sugar and metals), grain milling (e.g., wheat and rice), oilseed crushing, alloy preparation, energy generation, meat packing, and livestock production are examples of processes that comply with this Leontief function. In the farm sector, feedlot, hog and poultry production are but some of the production processes that can be modeled by this function with reasonable accuracy.

Diewert (1971) has shown that the cost function dual to (2.24) is

$$(2.25) \quad C = \Phi\, s_t\, Q_t - c(Q_t; v_t)$$

where C is variable cost, s_t is material input price, $c(\cdot)$ is a convex nonmaterial cost function such that $c'(\cdot) > 0$, and v_t is a vector of nonmaterial input prices. We will assume that nonmaterial input prices are constant, and we will simply write $c(Q_t)$ instead of $c(Q_t; v_t)$ because we will not be concerned with nonmaterial input prices. Assuming that material input price is stochastic while nonmaterial input prices are constant is not as unrealistic as it may at first appear. This is because in many situations the largest share of variable cost is due to the material input. In addition, nonmaterial input prices are generally less volatile, and substitability among nonmaterial inputs should cause variable cost changes far less pronounced than those due to material input price changes. Therefore, the cash flow corresponding to a nonstoring firm with the Leontief production function (2.24) can be represented by

$$(2.26)\quad \pi_t = p_t\, Q_{t-1} - \Phi\, s_t\, Q_t - c(Q_t) \quad \text{s.t.} \quad Q_t \geq 0$$

Comparing (2.26) with (2.2) reveals that the latter is a special case of the former, in which $\Phi = 1$, $s_t = p_t$, and $I_{t+1} = Q_t = Q_t^S/\Phi$.

With random final product and material input prices, the solution to the optimization problem for this firm is given by[12]

$$(2.27)\quad M_t(W_T, e_t) = \max_{Q_t \geq 0} £_t(W_T) | e_t$$

where: $£_T(W_T) = U(W_T)$

$$£_t(W_T) = \int_{p_{t+1}} \int_{s_{t+1}} M_{t+1}(W_T, e_{t+1})$$

$$g_{t+1}(p_{t+1}, s_{t+1} | p_0, s_0, \ldots, p_t, s_t)\, ds_{t+1}\, dp_{t+1}, \; 0 \leq t < T$$

The cash flows in expression (2.27) are as shown in (2.26), and $g_{t+1}(p_{t+1}, s_{t+1} | p_0, s_0, \ldots, p_t, s_t)$ is the conditional density function of s_{t+1} and p_{t+1} given $(p_0, s_0, \ldots, p_t, s_t)$. It is clear that optimal production at terminal date T is zero ($Q_T = 0$), and that the Kuhn-Tucker condition corresponding to any previous date is

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[12]As it will become more clear in Chapter IV, this firm at date t must decide how much to sell from what produced at the preceding date (Q_{t-1}), how much to produce for sale at next date (Q_t), and how much material input to use (Q_t^S). But the firm will always sell all beginning stocks as long as current output price is positive and it cannot store, and optimal material input use is given by $Q_t^S = \Phi\, Q_t$. Hence, the decision variable set reduces to only Q_t.

$$(2.28)\quad \frac{\partial \pounds_t}{\partial Q_t} = r_{t+1} \dots r_{T-1} [\mathbb{E}_t(p_{t+1} M_{t+1}') - r_t (\Phi\, s_t + c')\, M_t'] \leq 0,\ Q_t \geq 0,\ Q_t \frac{\partial \pounds_t}{\partial Q_t} = 0$$

With this basic setting we are in conditions of deriving a set of results analogous to Propositions 2.3 and 2.4 for the productive firm, namely, Propositions 2.5 and 2.6. Note that in Proposition 2.6 we use the following expressions regarding the relationship between output and material input prices:

$$(2.29)\quad s_t = \alpha\, p_t + u_t$$

$$(2.30)\quad s_t = \beta\, (p_t)^{\gamma}\, w_t$$

where α, β, and γ are positive constants, u_t is a random variable independent from p_t, and w_t is a positive random variable independent from p_t.

Proposition 2.5: Myopic reservation price for production The reservation price above which a myopic risk-averse firm does not produce is equal to the risk-neutral reservation price. A myopic risk-averse firm will produce at a level where discounted expected next-date output price is higher than weighted current material input price plus marginal production cost.

Proof. See Appendix A.

Corollary to Proposition 2.5 The myopic risk-averse firm produces less than the risk-neutral firm.

<u>Proposition 2.6: Forward-looking reservation price for production</u> (1) If output and material input prices are positively related, the reservation price above which a forward-looking risk-averse firm does not produce is generally different from the risk-neutral or the myopic risk-averse reservation price. Moreover, such a firm does not necessarily produce an amount at which discounted expected next-date output price is higher than weighted current material input price plus marginal production cost.

(2) If the firm is CARA and if output and material input prices are related as in (2.29) or (2.30). Then, the forward-looking reservation price is higher than the risk-neutral or the myopic risk-averse reservation price if (a) next-date output price is independently distributed from all posterior prices, or (b) the decision maker is sufficiently absolute risk-averse, or (c) output price follows a stationary autoregressive process and the decision maker is sufficiently forward-looking.

Proof. See Appendix A.

<u>Corollary to Proposition 2.6</u> If output and material input prices are related as in (2.29) or (2.30) and conditions (a), (b), or (c) in Proposition 2.6 are met, there exists a range of current prices over which the CARA forward-looking firm produces more than the risk-neutral one.

The intuition for Propositions 2.5 and 2.6 is the same as for the speculative storing firm. Again, our findings extend and qualify the standard results of the firm under uncertainty. For example, Proposition 2.6 explains the real-world observation that in many instances firms continue producing even if they expect not to recover their variable costs over short periods of time.

The key to the behavioral hypotheses derived for the forward-looking CARA firm is the positive contemporaneous relationship between output and material input prices. The obvious question that arises is how strong and of what sign is that relationship in real-world situations. To this end, we report in Table 2.2 the correlation coefficients for six pairs of contemporaneous output and material input prices belonging to the U.S. agricultural sector. It can be seen that in all cases output and input prices bear a positive relationship. Table 2.2 also shows that the output-material input price relationship may be too strong to be neglected a priori when analyzing the firm under uncertainty. This is so even for firms in the farm sector, as suggested by the high correlation coefficients between the price pairs slaughter steers-feeder steers, egg-feed, and broiler-feed.

Our results have implications for empirical work. First, the usual technique of a priori restricting the firm's production response to risk to be the same for all production levels may be inappropriate. In fact, doing so may bias empirical results towards rejection of the hypothesis that risk affects firm behavior. This observation is supported by empirical studies reporting that output price variance has a relatively low impact on production (e.g., Brorsen et al. 1985; Antonovitz and Roe 1986; Brorsen, Chavas, and Grant 1987; Aradhyula and Holt 1989; Antonovitz and Green 1990), and that material input price has a relatively higher effect on production than the expected output price (e.g., Antonovitz and Roe, Antonovitz and Green). Second, relaxing the myopic constraint seems relevant given the recent developments done towards allowing for both rational expectations and risk aversion (Aradhyula and Holt, Antonovitz and Green). Even though forward-looking behavior is not synonymous of rational expectations, the concept of rational expectations seems much more consistent with forward-looking than with myopic behavior.

Our findings are also relevant for the study of business cycles. Forward-looking CARA firms tend to produce less than risk-neutral ones at high output levels, but more at low

Table 2.2. Coefficients of correlation between contemporaneous output and material input prices, 1976:1-1987:12

Output	Material Input	Coefficient of Correlation[a]
Wholesale beef	Live beef	0.985**
Meal	Soybeans	0.942**
Oil	Soybeans	0.859**
Slaughter steers	Feeder steers	0.908**
Eggs	Egg feed	0.751**
Broilers	Broiler grower feed	0.550**

[a]The coefficients were estimated using monthly data deflated by the Producer Price Index.

**Significant at 1%.

Wholesale beef and live beef: Average prices of choice yield grade 3 steers at leading marketing areas (Source: USDA).

Soybeans: Price of No.1 Yellow, Illinois processors (Source: USDA).

Meal: Price of 44 percent protein, bulk, FOB Decatur (Source: USDA).

Oil: Price of crude, tanks, FOB Decatur (Source: USDA).

Slaughter steers: Price of choice slaughter steers, 900-1,100 pounds, Omaha (Source: USDA).

Feeder steers: Price of medium frame number one feeder steers, 600-700 pounds, Kansas City (Source: USDA).

Eggs and broilers: Prices received by farmers (Source: Weimar and Cromer 1990).

Egg feed: Egg feed costs (Source: Weimar and Cromer 1990).

Broiler grower feed: Prices paid by farmers (Source: Weimar and Cromer 1990).

levels of production. This means that forward-looking CARA firms will dampen the effects of business cycles.

Conclusions

A well-known result from the theory of the firm under uncertainty is that a myopic risk-averse firm produces less than an otherwise identical risk-neutral one. Our analysis reveals, however, that this conclusion is due to the assumption of myopic behavior and/or lack of correlation between output and material input prices. If output and material input prices are correlated, a risk-averse forward-looking firm may produce more or less than a risk-neutral one.

We show that if the forward-looking firm exhibits constant absolute risk aversion and if material input prices are positively related to output prices, the risk averse firm will produce less than a risk-neutral one at some prices if at least one of the following conditions apply: (a) the next-date output price is independently distributed from all posterior prices, (b) the decision maker is sufficiently risk averse, or (c) the output price follows a stationary autoregressive process and the decision maker is sufficiently forward-looking. In such instance, risk-averse production exceeds risk-neutral output at low levels of production, and the opposite is true at high production levels.

The model introduced in this chapter provides a rationale for stylized facts in microeconomics. For example, it explains why firms continue producing (or storing) in the short run even at an expected loss, and why farmers spread sales over time as a means to reduce risk. Our findings may also explain why empirical studies have found that the price variance has a relatively small impact on production.

CHAPTER III. COMPETITIVE FIRMS IN THE PRESENCE OF FORWARD MARKETS

With rare exceptions, previous work on hedging behavior has assumed a single production cycle. This implicitly assumes that the firm is myopic because such firm is not concerned about events that occur after the end of the current production cycle. This assumption has been carried over from the risk and uncertainty literature and can be justified on the basis of simplicity. Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) applied Sandmo's model of the myopic firm under uncertainty to analyze the behavior of the firm in the presence of a forward market for output. They showed that the competitive risk-averse firm separates production from hedging decisions. They also proved that it is optimal to place a full hedge (i.e., to short hedge the entire production) if the forward price is unbiased. Otherwise, it is optimal to short hedge more (less) than total output when the forward price is greater than (less than) the expected cash price.

A straightforward consequence of full-hedge optimality under unbiased forward prices is that most farmers should place full hedges most of the time. This should be the case because there is empirical evidence that futures prices are not significantly biased (Gray 1961, Rockwell 1967, Tomek and Gray 1970, Just and Rausser 1981). But not all farmers hedge all of their output. Extensions to the myopic hedging model have been proposed that would explain this behavior. These include the introduction of production risk (Chavas and Pope 1982, Losq 1982, Honda 1983, Grant 1985), basis risk (Batlin 1983, Paroush and Wolf 1989), hedging costs (Chavas and Pope), hedging restrictions (Antonovitz and Roe 1986, Antonovitz and Nelson 1988), and imperfect markets (Katz 1984).

There may be instances, however, where the myopic assumption itself leads to faulty conclusions about optimal hedging behavior. Consider for example a risk-averse livestock feeder who plans to remain in production beyond the current feeding cycle. At the beginning of the feeding cycle, the producer faces revenue risk because end-of-cycle output price is uncertain. But the firm also plans to purchase additional feeder cattle for the subsequent production cycle. For example, a beef fattener may plan to simultaneously sell fat cattle and purchase feeder cattle at the end of this feeding cycle. Revenue risk in this case will include output and input price risks. In most cases these risks will tend to offset each other because feeder cattle prices tend to be higher when fat cattle prices are high, and vice-versa. Intuitively, it is clear that the optimal hedge for a forward-looking firm (who plans to remain in production) will be different from a firm who myopically considers only output price risk.

Forward-looking hedging behavior was analyzed by Anderson and Danthine (1983), and by Hey (1987). Anderson and Danthine allow the firm to revise its hedging decisions during the production cycle but assume a single production cycle. They show that forward-looking producers should separate production and hedging decisions, but that if futures price is unbiased producers should not hedge all of their output. Hey allows for more than one production cycle and also finds that separation and suboptimality of full hedging hold. Hey's model is different from the one developed here because he assumes that (a) intertemporal utility is additive, (b) output cash prices are independently distributed and follow a constant distribution, and (c) sales decisions are taken after production and hedging decisions rather than simultaneously. Hey's results depend crucially on the sequential timing he imposes on sales, production, and hedging decisions.

The purpose of this chapter is to formally demonstrate the concept that the forward-looking optimal hedge is different from the myopic optimal hedge. We postulate a risk-averse firm that maximizes expected utility of terminal wealth, and derive some propositions regarding optimal hedging behavior under both myopic and forward-looking hypotheses. Unless stated otherwise, we retain the basic assumptions made in Chapter II. Because the correlation between output and input prices is more clear for speculative storage, we first present results for the speculative firm who only stores and then for the firm who is involved in production and cannot store. The last section reports the main conclusions from the chapter.

A Theoretical Model of Speculative Storage

We first assume the firm's productive activity is the speculative storage of a particular product, and that the firm can trade in a forward market for this product.[13] We hypothesize that at any date t there are only two positions that can be negotiated in the forward market: one for delivery at t+1, and the other for immediate delivery (i.e., delivery at t).[14] We denote by F_t the net short forward position for delivery at time t+1 open at date t. There are no restrictions on the amount or sign of the forward position held by the firm, except that the firm cannot have an open position for delivery at date T+1 at the end of the terminal trading date ($F_T = 0$), and that it cannot hold an open position for delivery at time t at the end of trading date t ($F_{t,t} = - F_{t-1,t}$, where the first subscript denotes the opening date and the second one the delivery date).[15] The cash flow from opening a forward contract lags by one period because forward trades do not involve cash flows until open positions are closed. The forward price prevailing at t for immediate delivery is identical to the current cash price (p_t). The forward

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[13]By using a forward instead of a futures market we can ignore basis risk. This facilitates the analysis to a great extent.

[14]We do not require actual delivery, but we still use this term for clarity of exposition. Forward commitments may be canceled either by delivering the good or by undertaking an opposite transaction in the forward market.

[15]This means that at any date t < T firms have only one free choice regarding the two tradable positions in the forward market, which is how much to commit for delivery (or receipt) at t+1. The other activity is to cancel out the open position for delivery at t, which is not a free choice because it must be done to satisfy the restrictions.

price at t for delivery in the following date t+1 (denoted by f_t), however, will be generally different from the current cash price p_t.

Under the above specifications, the firm's cash flow at any date $t \leq T$ is represented by

$$(3.1) \quad \pi_t = p_t P_t - i(I_t - P_t) + (f_{t-1} - p_t) F_{t-1} \qquad \text{s.t.} \quad I_{t+1} = I_t - P_t \geq 0$$

Note that we must now restrict $i(\cdot)$ to be *strictly* convex in order to obtain a bounded solution. We hypothesize that the firm selects the levels of current good sales (P_t) and current hedging (F_t) that maximize expected utility, given the available information (e_t). Hence, optimal decisions are made by solving the dynamic programming problem

$$(3.2) \quad M_t(W_T, e_t) = \max_{d_t} £_t(W_T) | e_t$$

where: $£_T(W_T) = U(W_T) + \eta_T (I_T - P_T)$

$$£_t(W_T) = \int_{p_{t+1}} \int_{f_{t+1}} M_{t+1}(W_T, e_{t+1})$$

$$h_{t+1}(p_{t+1}, f_{t+1} | p_0, f_0, \ldots, p_t, f_t)\, df_{t+1}\, dp_{t+1} + \eta_t (I_t - P_t), \; 0 \leq t < T$$

$$\mathbf{d}_t = (P_t, F_t) \text{ if } 0 \leq t < T, \; \mathbf{d}_T = (P_T, 0)^{16}$$

Terminal wealth is as defined in (2.1), and cash flows are given by (3.1). The function $h_{t+1}(p_{t+1}, f_{t+1} | p_0, f_0, \ldots, p_t, f_t)$ represents the conditional density function of p_{t+1} and f_{t+1}

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[16] Recall that $F_T = 0$ by assumption.

given $(p_0, f_0, \ldots, p_t, f_t)$. The vector $\mathbf{d}_t$ contains the firm's decision variables corresponding to date t. Applying the techniques employed in Chapter II, it is straightforward to show that the optimal decision vector corresponding to the terminal date is $\mathbf{d}_T = (I_T, 0)$, that terminal utility is maximized at

$$(3.3) \quad M_T(W_T, e_T) = U[r_{T-1} W_{T-1} + p_T I_T + (f_{T-1} - p_T) F_{T-1}]$$

and that the FOCs for dates previous to the terminal time $(0 \leq t < T)$ are

$$(3.4) \quad \frac{\partial \pounds_t}{\partial P_t} = r_{t+1} \cdots r_{T-1} [r_t (p_t + i') M_t' - E_t(p_{t+1} M_{t+1}')] - \eta_t = 0$$

$$(3.5) \quad \frac{\partial \pounds_t}{\partial F_t} = r_{t+I} \cdots r_{T-1} [f_t M_t' - E_t(p_{t+1} M_{t+1}')] = 0$$

$$(3.6) \quad \frac{\partial \pounds_t}{\partial \eta_t} = I_t - P_t \geq 0, \eta_t \geq 0, \eta_t \frac{\partial \pounds_t}{\partial \eta_t} = 0$$

Before proceeding, it is necessary to know the determinants of the optimal physical decisions (i.e., the variables that affect optimal storage I_1 or, equivalently, optimal sales P_0). The main results regarding this issue are summarized in Proposition 3.1.

<u>Proposition 3.1: Storage and sales behavior</u> In the presence of a forward market, optimal storage (or sales) for a risk-averse firm is determined independently from the subjective joint distribution of random variables, from the decision maker's degree of risk aversion, and from the optimal hedging decision. If positive, optimal storage is such that discounted current forward price equals current cash price plus marginal storage cost. These results hold for both myopic and forward-looking firms.

Proof. According to FOC (3.5), at the optimum the equality

$$(3.7)\quad E_0(p_1 M_1') = f_0 M_0'$$

holds. Substituting (3.7) into FOC (3.4) and rearranging yields

$$(3.8)\quad f_0 - r_0 [p_0 + i'(I_1)] = - \eta_0/(r_1 \cdots r_{T-1} M_0')$$

Hence:

a. If $f_0 < r_0 [p_0 + i'(0)]$, then $\eta_0 > 0$, and therefore $I_1 = 0$.

b. If $f_0 = r_0 [p_0 + i'(0)]$, then $\eta_0 = I_1 = 0$.

c. If $f_0 > r_0 [p_0 + i'(0)]$, then $\eta_0 = 0$, and therefore $I_1 > 0$ satisfying $f_0 = r_0 [p_0 + i'(I_1)]$.

Q.E.D.

Proposition 3.1 shows that separation between physical and hedging decisions is a robust property of the model. This is true because it holds for either myopic or nonmyopic behavior. Our results extend to the forward-looking scenario the findings by Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980), without the simplifying assumptions used by Hey (1987). Optimal storage (and sales) behavior is completely characterized in the proof to Proposition 3.1, and comparative statics follow easily from total differentiation of $f_0 - r_0 [p_0 + i'(I_1)] = 0$.[17]

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[17]Note that in the forward-looking scenario we cannot use jointly normally distributed prices to justify mean-variance analysis. Next-date storage (I_2) is random but cannot follow a normal distribution because firms do not store if $f_t < r_t [p_t + i'(0)]$.

We turn now to the focus of this chapter, i.e., the characterization of the optimal hedge. To this end, we will find useful to rewrite one of the components of FOC (3.5) in an alternative way, namely:

$$(3.9)\quad \mathbb{E}_0(p_1 M_1') = \mathbb{E}_0[p_1\, \mathbb{E}_0(M_1' | p_1)]$$

$$= \mathbb{E}_0(p_1)\, \mathbb{E}_0(M_1') + \mathrm{Cov}[p_1, \mathbb{E}_0(M_1' | p_1)]$$

where: $\mathbb{E}_0(M_1' | p_1) = \int_{f_1} M_1' f_1(f_1 | p_0, f_0, p_1)\, df_1 > 0$

By employing (3.9) and the fact that $\mathbb{E}_t(M_{t+1}') = M_t'$, we can state FOC (3.5) as follows:

$$(3.10)\quad [f_0 - \mathbb{E}_0(p_1)]\, M_0' = \mathrm{Cov}[p_1, \mathbb{E}_0(M_1' | p_1)]$$

Inspection of the sign of the covariance term in expression (3.10) allows us to state the results summarized in Propositions 3.2 and 3.3.

<u>Proposition 3.2: Myopic storage hedge</u> The optimal hedge for a myopic risk-averse firm that perceives the forward price to be unbiased is to (short) sell forward the total amount stored. This hedge is independent from the myopic firm's degree of risk aversion.

Proof. According to (3.10), at the optimum $[f_0 - \mathbb{E}_0(p_1)]$ and $\mathrm{Cov}[p_1, \mathbb{E}_0(M_1' | p_1)]$ must bear equal signs because $M_t' > 0$. In particular, $\mathrm{Cov}[p_1, \mathbb{E}_0(M_1' | p_1)] = 0$ if $f_0 = \mathbb{E}_0(p_1)$. For the myopic firm date $0 = T-1$, and from (3.3) we have $\mathbb{E}_{T-1}(M_T' | p_T) = M_T'$. Then,

$$(3.11)\quad \frac{\partial \mathbb{E}_{T-1}(M_T' \mid p_T)}{\partial p_T} = (I_T - F_{T-1})\, M_T'' \gtrless 0 \text{ as } F_{T-1} \lessgtr I_T$$

because $M_T'' < 0$. But p_T is monotonically increasing in p_T and $\mathbb{E}_{T-1}(M_T' \mid p_T)$ is monotonically increasing (decreasing) in p_T if $F_{T-1} > I_T$ ($F_{T-1} < I_T$). Hence, applying Theorem 43 in Hardy, Littlewood, and Pólya (1967) we obtain

$$(3.12)\quad \mathrm{Cov}[p_T, \mathbb{E}_{T-1}(M_T' \mid p_T)] \gtrless 0 \text{ as } F_{T-1} \lessgtr I_T$$

In particular, if $f_{T-1} = \mathbb{E}_{T-1}(p_T)$ it must be true that $F_{T-1} = I_T$. Q.E.D.

Proposition 3.3: Forward-looking storage hedge (1) The optimal hedge for a forward-looking risk-averse firm that perceives the forward price to be unbiased is not necessarily to sell forward the entire quantity stored. Furthermore, the optimal forward-looking hedge depends upon the firm's degree of risk aversion.

(2) If the firm is CARA and (a) next-date cash price is independently distributed from next-date forward price and from all posterior (cash and forward) prices, or (b) the decision maker is sufficiently absolute risk-averse, then the optimal forward-looking hedge under unbiased forward price is strictly smaller than the entire amount stored.

Proof. We show only part (2) of Proposition 3.3, because it implies part (1). For a nonmyopic CARA firm we have

$$(3.13)\quad \frac{\partial \mathbb{E}_0(M_1'|\,p_1)}{\partial p_1} = r_1 \ldots r_{T-1}\,(I_1 - F_0)\,\mathbb{E}_0(M_1''|\,p_1) - r_1 \ldots r_{T-1}\,\mathbb{E}_0(I_2\,M_1''|\,p_1)$$

$$- \int_{f_1} \frac{M_1''}{\lambda}\,\frac{\partial f_1(f_1|\,p_0, f_0, p_1)}{\partial p_1}\,df_1$$

$$- \int_{f_1} \{\max_{d_1}[\int_{p_2}\int_{f_2} \frac{M_2''}{\lambda}\,\frac{\partial h_2(p_2, f_2|\,p_0, f_0, p_1, f_1)}{\partial p_1}\,df_2\,dp_2]\}\,f_1(f_1|\,p_0, f_0, p_1)\,df_1$$

$$- \ldots - \int_{f_1} \{\max_{d_1}[\int_{p_2}\int_{f_2} \max_{d_2}(\int_{p_3}\int_{f_3} \ldots$$

$$\int_{p_{T-1}}\int_{f_{T-1}} \max_{d_{T-1}}(\int_{p_T}\int_{f_T} \frac{M_T''}{\lambda}\,\frac{\partial h_T(p_T, f_T|\,p_0, f_0, \ldots, p_{T-1}, f_{T-1})}{\partial p_1}\,df_T\,dp_T)$$

$$h_{T-1}(p_{T-1}, f_{T-1}|\,p_0, f_0, \ldots, p_{T-2}, f_{T-2})\,df_{T-1}\,dp_{T-1}) \ldots$$

$$h_2(p_2, f_2|\,p_0, f_0, p_1, f_1)\,df_2\,dp_2]\}\,f_1(f_1|\,p_0, f_0, p_1)\,df_1$$

where $f_1(f_1|\,p_0, f_0, p_1)$ is the conditional density function of f_1 given (p_0, f_0, p_1). Using the fact that $M_{t+1}'' < 0$, we get[18]

$$(3.14)\quad (I_1 - F_0)\,\mathbb{E}_0(M_1''|\,p_1) \gtrless 0 \text{ as } F_0 \lessgtr I_1$$

$$(3.15)\quad \mathbb{E}_0(I_2\,M_1''|\,p_1) < 0$$

Under condition (a), all terms in the right-hand side of (3.13) vanish except the first two terms. Then $F_0 \geq I_1$ implies $\partial \mathbb{E}_0(M_1'|\,p_1)/\partial p_1 > 0$ and therefore

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[18]The proof that $\mathbb{E}_t(I_{t+2}\,M_{t+1}''|\,p_{t+1}) < 0$ is shown in Appendix B.

$Cov[p_1, \mathbb{E}_0(M_1' | p_1)] > 0$. Hence, if $f_0 = \mathbb{E}_0(p_1)$ we must have $F_0 < I_1$.

The proof for condition (b) is straightforward, by noting that

$$(3.16)\ \lim_{\lambda \to \infty} \frac{\partial \mathbb{E}_0(M_1' | p_1)}{\partial p_1} = r_1 \ldots r_{T-1} (I_1 - F_0) \mathbb{E}_0(M_1'' | p_1)$$

$$- r_1 \ldots r_{T-1} \mathbb{E}_0(I_2 M_1'' | p_1) \quad \text{Q.E.D.}$$

Corollary to Proposition 3.3: CARA forward-looking storage hedge If the firm is CARA and (a) next-date cash price is independently distributed from next-date forward price and from all posterior (cash and forward) prices, or (b) the decision maker is sufficiently absolute risk-averse, then the optimal forward-looking hedge may be strictly smaller than the entire amount stored under an upwardly biased forward price.

The results reported in Proposition 3.2 are analogous to those obtained by Holthausen (1979), and Feder, Just, and Schmitz (1980), and demonstrate that this model is consistent with the standard literature. Our findings about the optimal forward-looking hedge reveal that full-hedge optimality under unbiased forward price is not robust because it applies only to the myopic scenario. From the proofs of Propositions 3.2 and 3.3, it is clear that the simplicity of the optimal myopic hedge under unbiased forward price is attributable to the fact that the myopic firm assumes with certainty that whatever it stores now will be completely sold in the next trading time, and that it will store nothing at the next date (i.e., $I_{2=T+1} = 0$). Also, the myopic firm plans to hedge nothing at the next trading time (i.e., $F_{1=T} = 0$). In contrast, the forward-looking firm assigns a positive probability to storing and/or a nonzero hedging at the next trading date. But next-date storage and hedge are correlated with next-date cash price, and therefore they serve as (partial) substitutes for current hedging. It is this substitution effect that leads to full-hedge suboptimality in the forward-looking scenario.

An alternative interpretation of the full-hedge suboptimality result is that we have formalized a common behavioral pattern known as *anticipatory* hedging. The firm may operate in the forward market to speculate, and/or to place two types of hedges, namely *risk-avoidance* and anticipatory hedges.[19] If the forward price is unbiased, the firm does not speculate and trades forward only to hedge. The risk-avoidance hedge consists of selling current storage forward to reduce its price risk, whereas the anticipatory hedge is placed to avoid the price risk of next-date storage. Therefore, the risk-avoidance hedge is identical to current storage. In contrast, the size of the anticipatory hedge depends upon the distribution of (random) next-date storage and hedge, the agent's degree of risk aversion, and the joint distribution of random prices, among other factors. Hence, the sum of risk-avoidance and anticipatory hedges generally differs from current storage, and it depends upon the degree of risk aversion. This is true unless the firm currently knows exactly how much it will store and hedge at next-date, so that next-date storage and hedge are nonrandom. The myopic case is an example of the latter situation, in which next-date storage and hedge are known to be exactly zero ($I_{2=T+1} = 0$, $F_{1=T} = 0$).

The general suboptimality of the full hedge under unbiased forward prices is an important result. It is widely accepted that full hedging is optimal when forward price is unbiased. The full hedge is appealing because of its simplicity. Also, its normative content is easy and broadly applicable because it makes complete abstraction of the agent's degree of risk aversion. Our model brings attention to the fact that, despite these appealing characteristics,

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[19]Risk-avoidance and anticipatory hedges are defined in Marshall (1989).

full-hedge optimality depends crucially upon assuming myopic behavior or independence of output and material input prices.

Given the previous discussion, it is easier to understand why the full hedge overestimates the optimal forward-looking hedge under unbiased forward prices when the firm is CARA and assumption (a) in Proposition 3.3 applies. Next-date storage is negatively associated with next-date cash price, and therefore next-date storage eliminates part of next-date cash price risk: revenue from current storage will be low if p_1 is low, but then the firm will be able to buy material input to store at a low price, thereby partially offsetting the lower revenue. This means that next-date storage is an imperfect substitute for current hedging, so that the hedge required to minimize next-date cash price risk is smaller than it would be if next-date storage did not contribute to risk reduction.

If the next-date cash price is related to the next-date forward price, the optimal forward-looking CARA hedge may be larger than the amount stored even under unbiased forward prices. This may happen because next-date cash price indirectly affects the current hedge through its relationship with next-date forward price. The sign and magnitude of this indirect effect depends on the size of next-date hedge, which in turn may be positive or negative and large enough to yield a current hedge exceeding storage. If the forward-looking CARA firm is sufficiently risk averse, however, the direct effect of next-date cash price on current hedge outweighs any indirect effect, yielding an optimal hedge that is smaller than storage under unbiased forward prices.

Expression (3.13) is helpful in that it allows us to separate clearly the three main components involved in the optimal forward-looking hedge. The first term in the right-hand side of (3.13) is the risk-avoidance component, whereas the second and third terms are the anticipatory components. The risk-avoidance term vanishes if $F_0 = I_1$. The anticipatory component can be further divided into *direct* and *indirect* anticipation terms (i.e., the second and third right-hand side terms, respectively). The direct anticipation component is due to the effect of next-date storage (I_2). The direct anticipation term is strictly positive irrespective of risk attitudes or price distributions, and requires a long hedge ($F_0 < 0$) to equal zero. Finally, the indirect anticipation component involves the impact on current hedging attributable to the interaction between the risk attitude and the price distribution, and it has an ambiguous sign.

When assumptions (a) or (b) of Proposition 3.3 hold, the optimal forward-looking hedge under unbiased forward price is strictly negative if nothing is stored (e.g., $F_0 < I_1 = 0$), so that the nonmyopic firm establishes a long forward position. In contrast, the optimal myopic hedge in the same situation is $F_0 = 0$. This is a useful result, because it explains the existence of anticipatory hedging under unbiased forward prices without resorting to ad-hoc assumptions.

In the standard myopic framework, anticipatory hedging is modeled assuming that the firm currently knows exactly how much it will store and hedge at next date, which is clearly an inconsistent hypothesis. If the firm is myopic, we have shown that it is suboptimal to expect next-date sales to be anything less than beginning stocks. If the firm is nonmyopic but knows next-date storage and hedge with certainty, then either prices are nonstochastic, or the firm does not behave optimally.

A Productive Non-Storing Firm

Having analyzed the hedging firm involved with storage only, we will address now the case in which the firm produces and does not store for speculative purposes. We will assume that the production and cost functions are given by expressions (2.24) and (2.25), respectively, but we will restrict the nonmaterial cost function $c(\cdot)$ to be strictly convex to obtain a bounded solution. Because in this instance output and material input prices are different from each other, to make the analysis more interesting we will hypothesize that there exist forward markets for both output and material input. We will denote the forward price and forward position corresponding to material input by f_t^S and F_t^S, respectively. Then, the cash flow for the productive nonstoring firm becomes

$$(3.17)\quad \pi_t = p_t\, Q_{t-1} - \Phi\, s_t\, Q_t - c(Q_t) + (f_{t-1} - p_t)\, F_{t-1} + (f_{t-1}^S - s_t)\, F_{t-1}^S \qquad \text{s.t.}\quad Q_t \geq 0$$

and the dynamic programming problem to obtain the optimal solution is

$$(3.18)\quad M_t(W_T, e_t) = \max_{d_t} \pounds_t(W_T) \mid e_t$$

where: $\pounds_T(W_T) = U(W_T)$

$$\pounds_t(W_T) = \int_{p_{t+1}} \int_{s_{t+1}} \int_{f_{t+1}} \int_{f_{t+1}^S} M_{t+1}(W_T, e_{t+1})$$

$$j_{t+1}(p_{t+1}, s_{t+1}, f_{t+1}, f_{t+1}^S \mid p_0, s_0, f_0, f_0^S, \ldots, p_t, s_t, f_t, f_t^S)$$

$$df_{t+1}^S\, df_{t+1}\, ds_{t+1}\, dp_{t+1},\ 0 \leq t < T$$

$$\mathbf{d}_t = (Q_t, F_t, F_t^S) \text{ if } 0 \leq t < T,\ \mathbf{d}_T = (Q_T, 0, 0),\ Q_t \geq 0$$

Terminal wealth and cash flows are given by (2.1) and (3.17), respectively. The function $j_{t+1}(p_{t+1}, s_{t+1}, f_{t+1}, f_{t+1}^S \mid p_0, s_0, f_0, f_0^S, \ldots, p_t, s_t, f_t, f_t^S)$ represents the conditional density of p_{t+1}, s_{t+1}, f_{t+1}, and f_{t+1}^S given $(p_0, s_0, f_0, f_0^S, \ldots, p_t, s_t, f_t, f_t^S)$.

The analysis of optimal production and hedging for a firm with a cash flow described by (3.17) can be performed using similar procedures as before. To avoid repetition, we outline the main results here, and focus on the most important behavioral differences between speculative storing and productive nonstoring firms. The maximum attainable utility at the terminal date can be shown to be

$$\text{(3.19)}\quad M_T(W_T, e_T) = U[r_{T-1}\, W_{T-1} + p_T\, Q_{T-1} + (f_{T-1} - p_T)\, F_{T-1} + (f^S_{T-1} - s_T)\, F^S_{T-1}]$$

and the FOCs for any date preceding the terminal time are

$$\text{(3.20)}\quad \frac{\partial \pounds_t}{\partial Q_t} = r_{t+1} \cdots r_{T-1}\, [\mathbb{E}_t(p_{t+1}\, M_{t+1}') - r_t\, (\Phi\, s_t + c')\, M_t'] \leq 0,\ Q_t \geq 0,\ Q_t\, \frac{\partial \pounds_t}{\partial Q_t} = 0$$

$$\text{(3.21)}\quad \frac{\partial \pounds_t}{\partial F_t} = r_{t+1} \cdots r_{T-1}\, [f_t\, M_t' - \mathbb{E}_t(p_{t+1}\, M_{t+1}')] = 0$$

$$\text{(3.22)}\quad \frac{\partial \pounds_t}{\partial F^S_t} = r_{t+1} \cdots r_{T-1}\, [f^S_t\, M_t' - \mathbb{E}_t(s_{t+1}\, M_{t+1}')] = 0$$

The most important results regarding the productive nonstoring firm are obtained by means of FOCs (3.20) through (3.22). These results are summarized as Propositions 3.4, 3.5, and 3.6, which are the respective counterparts of Propositions 3.1, 3.2, and 3.3.

<u>Proposition 3.4: Production behavior</u> In the presence of an output forward market, optimal production for a nonstoring risk-averse firm is independent from the subjective joint distribution of random variables, from the decision maker's degree of risk aversion, and from the optimal hedging decision. If positive, optimal production is such that discounted

current output forward price equals (weighted) current material input cash price plus marginal production cost. These results hold for both myopic and forward-looking firms.

Proof. See Appendix B.

Proposition 3.5: Myopic production hedge The optimal hedge for a myopic nonstoring risk-averse firm that perceives output and material input forward prices to be simultaneously unbiased is to sell the entire production in the output forward market, and to sell nothing in the material input forward market. This hedge is independent from the myopic firm's degree of risk aversion.

Proof. See Appendix B.

Proposition 3.6: Forward-looking production hedge (1) The optimal hedge for a productive nonstoring forward-looking risk-averse firm is generally different from the optimal myopic hedge. Furthermore, the optimal forward-looking hedge depends upon the firm's degree of risk aversion.

(2) If the productive nonstoring forward-looking firm is CARA and (a) next-date output and material input cash prices are each independently distributed from all other contemporaneous and posterior prices, or (b) the decision maker is sufficiently absolute risk-averse, then the optimal hedge under unbiased output and material input forward prices consists of selling the entire production in the output forward market, and buying forward contracts of material input.

Proof. See Appendix B.

Proposition 3.4 confirms the robustness of the separation result, showing that it applies to a cost function which characterizes many production processes even when the firm is forward-looking. It is also important to note that separation holds irrespective of the existence of a forward market for material input. Propositions 3.5 and 3.6 highlight the differences between myopic and forward-looking hedging behaviors and confirm the weakness of the full-hedge optimality result.

Even for the myopic case, it will generally be true that $F_0 \neq Q_0$ and $F_0^s \neq 0$ simultaneously if output or material input forward prices (or both) are biased. This can be seen from[20]

$$(3.23)\quad \frac{\partial E_{T-1}(M_T' \mid p_T)}{\partial p_T} = (Q_{T-1} - F_{T-1})\, E_{T-1}(M_T'' \mid p_T)$$

$$- \int_{s_T} \int_{f_T} \int_{f_T^s} \frac{M_T''}{\lambda} \frac{\partial k_T(s_T, f_T, f_T^s \mid p_{T-1}, s_{T-1}, f_{T-1}, f_{T-1}^s, p_T)}{\partial p_T}\, df_T^s\, df_T\, ds_T$$

$$(3.24)\quad \frac{\partial E_{T-1}(M_T' \mid s_T)}{\partial s_T} = - F_{T-1}^s\, E_{T-1}(M_T'' \mid s_T)$$

$$- \int_{p_T} \int_{f_T} \int_{f_T^s} \frac{M_T''}{\lambda} \frac{\partial l_T(p_T, f_T, f_T^s \mid p_{T-1}, s_{T-1}, f_{T-1}, f_{T-1}^s, s_T)}{\partial s_T}\, df_T^s\, df_T\, dp_T$$

---

[20]Expressions (3.23) and (3.24) follow from (B6) and (B7) in Appendix B, respectively.

where $k_T(s_T, f_T, f^S_T | p_{T-1}, s_{T-1}, f_{T-1}, f^S_{T-1}, p_T)$ is the conditional density function of s_T, f_T, and f^S_T given $(p_{T-1}, s_{T-1}, f_{T-1}, f^S_{T-1}, p_T)$, and $l_T(p_T, f_T, f^S_T | p_{T-1}, s_{T-1}, f_{T-1}, f^S_{T-1}, s_T)$ is the conditional density function of p_T, f_T, and f^S_T given $(p_{T-1}, s_{T-1}, f_{T-1}, f^S_{T-1}, s_T)$. For example, if the material input forward price is biased and the output forward price is not, FOCs require $\partial \mathbb{E}_{T-1}(M_T' | s_T)/\partial s_T \neq 0$ and $\partial \mathbb{E}_{T-1}(M_T' | p_T)/\partial p_T = 0$. This will generally mean a nonzero forward position in the input market ($F^S_{T-1} \neq 0$) and an output hedge different from total production ($F_{T-1} \neq Q_{T-1}$). In fact, a full output hedge ($F_{T-1} = Q_{T-1}$) does not yield $\partial \mathbb{E}_{T-1}(M_T' | p_T)/\partial p_T = 0$ if $F^S_{T-1} \neq 0$ unless p_T and s_T are independently distributed.

Proposition 3.6 clarifies our previous explanations for the storage case. With unbiased forward prices, the optimum hedge consists of the risk avoidance hedge ($F_0 = Q_0$) and the anticipatory hedge ($F^S_0 < 0$). In terms of payoff with respect to alternative forward prices, the net effect of both forward positions (F_0, F^S_0) is similar to a less than fully hedged output position so long as output and input prices are correlated. Because of this, a predictable consequence of not having a forward market for material input is that the optimal forward-looking CARA hedge under unbiased forward price and conditions (a) or (b) is smaller than the entire production. This result is formalized in Proposition 3.7.

Proposition 3.7: Forward-looking hedge in the absence of input forward markets

Assume there is no forward market for material input, the productive nonstoring firm is CARA, and material input and output cash prices are related as in (2.29) or (2.30). Then, the optimal forward-looking hedge under unbiased forward price is smaller than the entire production if (a) next-date output cash price is independently distributed from next-date forward price and from all posterior prices, or (b) the decision maker is sufficiently absolute risk-averse.

Proof. See Appendix B.

Corollary to Proposition 3.7 If the assumptions in Proposition 3.7 are met, then the optimal forward-looking hedge may be strictly smaller than the entire amount stored under an upwardly biased forward price.

Proposition 3.7 reminds us that the standard full hedge optimality result depends crucially on the firm being myopic, or on output and material input cash prices being independent from each other. Full-hedge suboptimality under nonmyopic behavior and unbiased forward prices is important, and specially relevant for empirical work. Recently, some studies have been conducted to obtain empirical estimates of the "optimal hedge" when there is a futures rather than a forward market (Witt, Schroeder, and Hayenga 1987; Cecchetti, Cumby, and Figlewski 1988; Myers and Thompson 1989). The normative content of these studies is usually emphasized on the basis that the optimal hedge under unbiased futures prices is independent of the decision maker's degree of risk aversion (Batlin 1983; Benninga, Eldor, and Zilcha 1984). Our results suggest that this would be the case only if the agent is myopic, or output and material input cash prices are unrelated to each other.

Conclusions

In this chapter we have shown that separation between production and hedging is a robust result, because it holds even if firms are forward-looking. In the presence of forward markets, optimal production for a forward-looking firm is identical to an otherwise equivalent myopic firm. Optimal production is determined solely by nonrandom factors, and is independent from the agent's price expectations and from his degree of risk aversion.

In contrast, full-hedge optimality under unbiased forward price only holds if the decision maker is myopic or if output and material input cash prices are independent from each other. Full-hedging is suboptimal when the firm is forward-looking because in this instance the firm foresees that at next decision date it will stay in the market and it will take decisions based on the observed values of the relevant random variables. Hence, next-date decisions are random and affect the current risks faced by the firm, and therefore will have an impact on the optimal current hedge. If the forward-looking firm is CARA and (a) next-date cash prices are independent of other simultaneous and posterior prices, or (b) the decision maker is sufficiently risk-averse, then under unbiased forward price the optimal storage hedge is strictly smaller than the entire amount stored, and the optimal production hedge in the absence of a forward market for material input is strictly less than the quantity produced (assuming output and material input cash prices are positively related).

Our results may help explain why farmers are observed not to fully hedge, even when there is empirical evidence that futures prices are generally unbiased. Also, full-hedge suboptimality under nonmyopic behavior and unbiased forward prices appears relevant for studies concerned with the empirical estimation of optimal hedges in the presence of futures rather than forward markets.

CHAPTER IV. SHORT-RUN BEHAVIOR FOR A PRODUCTIVE STORING FIRM: THE CASE OF THE U.S. SOYBEAN-PROCESSING INDUSTRY

The objective function used in the theory of the firm typically contains variables reflecting output price times output quantity and a quantity-dependent cost function. Implicit in this approach are the assumptions that the firm simultaneously sells output and buys inputs at known prices. In many firms, however, much of the managerial effort is targeted toward buying inputs when their prices are lowest, selling output when its price is highest, using input and output storage to take advantage of price movements, and employing forward markets to hedge some of the risks associated with production and storage. These activities are particularly important in commodity-oriented firms, such as those involved in producing and processing food and natural resources.

In the medium and long term, these observed differences between production and output sales and between input purchases and usage are averaged out and seem trivial; however, the medium- and long-term behavior of the firm may reflect the cumulative impact of short-term decisions. In this case, a full understanding of long-term behavior will depend, in part, on how managers respond to short-term incentives.

In this chapter, we develop and test a theory of short-run competitive firm behavior under risk aversion in the presence of futures markets. Consistent with the preceding analysis, we allow the firm to be forward-looking. We also show how short-term parameters can be used to derive meaningful long-term response parameters. This chapter is organized as follows. In the following section we lay out the theoretical model. We then test some of the theoretical results using data from the U.S. soybean-processing industry, and we discuss our findings. In the final section we summarize the major conclusions of the chapter.

The Theoretical Model

Relaxation of the standard nonstorage constraint is one of the main contributions of our analysis. Hence, we make explicit allowance for storage of both output and material input. This means output sales and material input purchases will generally be different from the amount produced and the material input employed in the production process, respectively. We also allow for the presence of forward markets for both output and material input. This is the most general setting of our model; in situations where futures or forward markets are not available the more general scenario can be adjusted by omitting the relevant variables from the objective function. Therefore, given the specifications discussed in the preceding chapters, the particular form of the firm's cash flow at date t is represented by

$$\text{(4.1)} \quad \pi_t = p_t P_t - s_t S_t - c(Q_t^S/\Phi) - i(I_t - P_t) - i^S(I_t^S + S_t - Q_t^S)$$

$$+ (f_{t-1,t} - p_t) F_{t-1} + (f_{t-1;t}^S - s_t) F_{t-1}^S$$

$$\text{s.t.} \quad I_t = I_{t-1} - P_{t-1} + Q_{t-1} \geq P_t, \; I_t^S + S_t \geq Q_t^S = \Phi\, Q_t \geq 0$$

where S_t denotes material input purchases, $i^S(\cdot)$ is a strictly convex inventory cost function of material input $[i^{S\prime}(\cdot) > 0]$, and I_t^S is beginning inventory of material input at date t $[I_t^S = I_{t-1}^S + S_{t-1} - Q_{t-1}^S]$. Note that in this chapter we will use $f_{t-1,t}$ and $f_{t-1;t}^S$ to denote f_{t-1} and f_{t-1}^S, respectively, because it will be helpful to do so for the empirical application.

At any date t the firm chooses purchases and use of material input (S_t and Q_t^S), production ($Q_t = \Phi\, Q_t^S$), sales of final product (P_t), and hedging (F_t and F_t^S) so as to maximize expected utility of terminal wealth, given the information available (e_t). Hence, the optimal decisions at current date t = 0 are made by solving the dynamic programming problem

(4.2) $M_t(W_T, e_t) = \max_{d_t} £_t(W_T) | e_t$

where: $£_T(W_T) = U(W_T) + \eta_T (I_T - P_T) + \eta_T^S (I_T^S + S_T - Q_T^S)$

$£_t(W_T) = \mathbb{E}_t[M_{t+1}(W_T, e_{t+1})] + \eta_t (I_t - P_t) + \eta_t^S (I_t^S + S_t - Q_t^S),\ 0 \le t < T$

$\mathbf{d}_t = (P_t, Q_t^S, S_t, F_t, F_t^S)$ if $t < T$, $\mathbf{d}_T = (P_T, Q_T^S, S_T, 0, 0)$

η_t^S = Lagrangian multiplier corresponding to inventory of material input

and π_t is as defined in (4.1). The first-order conditions (FOCs) corresponding to this problem can be expressed as[21]

(4.3) $$\frac{\partial £_t}{\partial P_t} = r_{t+1} \cdots r_{T-1} [r_t (p_t + i') M_t' - \mathbb{E}_t(p_{t+1} M_{t+1}')] - \eta_t = 0$$

(4.4) $$\frac{\partial £_t}{\partial Q_t^S} = r_{t+1} \cdots r_{T-1} [\mathbb{E}_t(p_{t+1}/\Phi\, M_{t+1}') - r_t (s_t + q'/\Phi) M_t'] \le 0,\ Q_t^S \ge 0,\ Q_t^S \frac{\partial £_t}{\partial Q_t^S} = 0$$

(4.5) $$\frac{\partial £_t}{\partial S_t} = r_{t+1} \cdots r_{T-1} [\mathbb{E}_t(s_{t+1} M_{t+1}') - r_t (s_t + i^{S\prime}) M_t'] + \eta_t^S = 0$$

(4.6) $$\frac{\partial £_t}{\partial F_t} = r_{t+1} \cdots r_{T-1} [f_{t,t+1} M_t' - \mathbb{E}_t(p_{t+1} M_{t+1}')] = 0$$

(4.7) $$\frac{\partial £_t}{\partial F_t^S} = r_{t+1} \cdots r_{T-1} [f_{t;t+1}^S M_t' - \mathbb{E}_t(s_{t+1} M_{t+1}')] = 0$$

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[21]Expressions (4.3) through (4.5) are derived in Appendix C.

$$(4.8)\quad \frac{\partial \pounds_t}{\partial \eta_t} = I_t - P_t \geq 0,\ \eta_t \geq 0,\ \eta_t \frac{\partial \pounds_t}{\partial \eta_t} = 0$$

$$(4.9)\quad \frac{\partial \pounds_t}{\partial \eta_t^s} = I_t^s + S_t - Q_t^s \geq 0,\ \eta_t^s \geq 0,\ \eta_t^s \frac{\partial \pounds_t}{\partial \eta_t^s} = 0$$

The rationale for Kuhn-Tucker condition (4.4) is that the amount processed cannot be negative (i.e., production reversal is precluded). We have not added Kuhn-Tucker conditions to either (4.3) or (4.5)-(4.7) because we do not require non-negativity of final product sales, material input purchases, or forward positions. This means that the firm can buy final product, sell material input, or have a net long forward position.

The preceding FOCs can be further manipulated to yield separation between "physical" decisions (i.e., purchases, production, and sales) and hedging. This is readily shown by substituting (4.6) into (4.3) and (4.4), and (4.7) into (4.5), and rearranging, which yields the set of expressions (4.10)-(4.12) as an alternative to (4.3)-(4.5):

$$(4.10)\quad f_{t,t+1} - r_t\,[p_t - i'(I_t - P_t)] = \eta_t/(r_{t+1} \cdots r_{T-1}\, M_t')$$

$$(4.11)\quad f_{t,t+1}/\Phi - r_t\,[s_t - q'(Q_t^s/\Phi)/\Phi] \leq 0,\ Q_t^s \geq 0,\ Q_t^s\,\{f_{t,t+1}/\Phi - r_t\,[s_t - q'(Q_t^s/\Phi)/\Phi]\} = 0$$

$$(4.12)\quad f_{t;t+1}^s - r_t\,[s_t - i^{s\prime}(I_t^s + S_t - Q_t^s)] = \eta_t^s/(r_{t+1} \cdots r_{T-1}\, M_t')$$

Expression (4.11) allows us to solve for the optimal level of material input use (Q_t^s) independently from hedging, purchases, sales, and beginning inventories. A careful look at (4.10) and (4.8) reveals that output sales (P_t) are independent not only from the amounts hedged but also from use, purchases, and beginning stocks of material input. Output sales can take any value that does not exceed beginning stocks of final product. If sales equal beginning stocks, then $P_t = I_t$; if sales are strictly less than beginning stocks then $\eta_t = 0$, and the precise level of sales is obtained from (4.10). Similarly, expressions (4.12) and (4.9) allow us to solve for the optimal level of material-input purchases (S_t) independently from sales and beginning stocks of final product.

In summary, *the existence of forward markets for final product and material input leads to separation of purchases/processing/sales and speculative decisions for the forward-looking risk-averse firm.* Moreover, optimal purchases, processing, and sales are independent of the agent's degree of risk aversion and the distributions of random cash prices. Sales of final product are obviously independent of the level of risk aversion and random prices so long as sales equal beginning inventories (i.e., $P_t = I_t$). Alternatively, if sales of final product are smaller than beginning inventories (i.e., $P_t < I_t$), then the terms in which the risk attitude and the random prices appear collapse to zero, and again sales are independent of these variables. A similar analysis can be applied to show that purchases of material input are also independent of the decision maker's degree of risk aversion and the distribution of cash prices.

Comparative statics corresponding to output sales and to purchases and use of material input for an interior solution can be obtained by setting the right-hand terms in (4.10) through (4.12) equal to zero and totally differentiating the resulting expressions. This derivation is straightforward after recalling the properties imposed on the nonmaterial and storage cost functions [$c(\cdot)$, $i(\cdot)$, and $i^S(\cdot)$, respectively]. Comparative statics are summarized in Table 4.1.

The theoretical results reported in Table 4.1 indicate that use of material input should be negatively related to its current cash price and positively related to the forward price of final product. Beginning stocks of final product may or may not affect material input use, depending upon the particular nonmaterial cost function chosen. By imposing some constraints on the nonmaterial cost function we can obtain the intuitively appealing theoretical

Table 4.1. Theoretical effect of exogenous variables on material input use (production), material input purchases, and output sales over the decision horizon

Explanatory Variables	Endogenous Variables		
	Mat. Input Use	Mat. Input Purchases	Output Sales
Cash prices: input	–	–	0
output	0	0	+
Forward prices: input	0	+	0
output	+	+	–
Interest rate	–/?	–/?	+/?
Beginning stocks: input	0	–	0
output	0/–	0/–	+

result that material input use adjusts negatively to higher beginning stocks of final product.[22] The impact of the interest rate on input use is negative if use is independent of output beginning stocks, and it is ambiguous otherwise. Purchases of material input respond in the same fashion as material input usage but are also positively related to the current forward price of material input and negatively related to the beginning stock of material input. Sales of final product are independent of cash and forward prices of material input as well as beginning inventories of material input. Output sales are positively related to current output cash price and beginning inventories of final product, and they are negatively associated to current output forward price. The interest rate has a positive effect on sales if input use does not depend on output beginning stocks, and an ambiguous effect otherwise.

The existence and direction of the causal relationships summarized in Table 4.1 are very different from those predicted by the standard myopic model. This is true because in the myopic model processing, purchases, and sales are either identical or bear fixed relationships. It is interesting therefore to see if the hypothesized relationships of Table 4.1 are supported by an appropriate data set. This is the purpose of the remainder of the chapter.

Empirical Results and Discussion

We chose the U.S. soybean-processing industry to test our theoretical propositions because there are highly liquid futures markets for both material input (soybeans) and final goods (soyoil and soymeal) in the Chicago Board of Trade (CBOT). In addition, there are

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[22]This response to beginning inventories of final good is obtained by letting $q(\cdot) = q(Q_t^S/\Phi, I_t)$, $q_2 > 0$, $q_{22} > 0$, $q_{12} = 0$.

available high-quality data at a monthly frequency, which is the *observation horizon* we employed.[23]

Before turning to the description of the methodology, data, and estimation procedures, it is worthwhile to summarize the empirical results from the econometric model in terms comparable to Table 4.1. This inversion of the standard presentation procedure allows a more direct linkage of theory and practice and is justified in part by the necessary complexity of the application of the model. Table 4.2 is entirely analogous to Table 4.1, but it contains the estimated partial elasticities corresponding to the U.S. soybean-processing industry.[24] A comparison of Tables 4.1 and 4.2 demonstrates that use and purchases of material input, as well as final product sales generally follow the hypothesized pattern. The only exception is that beginning output stocks have a nonsignificant effect on material input purchases. Price variables have low or no significance in the input purchase equation which, however, is due to multicollinearity (this point is discussed in more detail in the next section).

The most important feature of these results is that decisions regarding input use, input purchases, and output sales can be treated separately and in a predictable way when we build models of the short-run behavior in these industries. In results presented later we show that the relationships left blank in Table 4.2 are nonsignificant. In the absence of the preceding theoretical analysis the lack of significance of these missing variables might seem counterintuitive. For example, one might (as the USDA does) use cash prices of oil and meal relative to cash price of soybeans as a measure of processing profitability (USDA, Economic and Statistics Service, Fats and Oils--Outlook and Situation). A priori, any of the endogenous

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[23]Observation horizon is "the length of time between successive observations of the data by the researcher" (Merton, 1982, p. 656).

[24]Soybean processors produce meal and oil in fixed proportions, and so there are two relevant output prices.

Table 4.2. Empirical estimates of the average partial elasticities of monthly material input use (production), material input purchases, and output sales of U.S. soybean processors with respect to selected exogenous variables, 1965:9-1986:12

Explanatory Variables	Endogenous Variables			
	Mat. Input Use	Mat. Input Purchases	Output Sales: Oil	Output Sales: Meal
Cash prices: soybeans	-0.36**	-1.91*		
oil			0.50**	
meal				0.12**
Futures prices: soybeans		0.63		
oil	0.134**	0.48*	-0.50**	
meal	0.226**	0.81*		-0.12**
Interest rate	-0.0065**	-0.03*	0.009**	0.002**
Beginning stocks: soybeans		-0.241**		
oil	-0.0256**	0.015	0.115**	
meal	-0.052**	-0.044		0.0333**

*Significant at 5%.

**Significant at 1%.

variables could be used as a measure of the *activity* of the firm, and any one or set of the explanatory variables as the incentives to which the firm responds.

To emphasize the differences in relative magnitudes among input use, input purchases, and output sales, in Table 4.3 we present short-term total elasticities of these variables with respect to prices. Table 4.3 differs from Table 4.2 in that the former includes the indirect effect of prices through the impact of input use (production) on input purchases (output sales).[25] The magnitudes of the total elasticities are directly comparable across the endogenous variables and show, for example, that soybean cash price causes a much greater change in soybean purchases than in soybean use. These elasticities indicate that soybean cash price is in fact the single most important factor affecting processors' behavior. Table 4.3 also indicates that in the short term processors adjust to changes in cash and futures prices mainly through their soybean purchases.

Monthly elasticities of purchases and sales with respect to own cash prices are larger in absolute value than the analogous elasticities with respect to futures. For soybean purchases this happens because soybean cash price affects profitability of both storage and crushings, while soybean futures influence only returns of soybean storage. The explanation for oil and meal sales is that futures have not only a direct impact on sales but also an opposite indirect effect through their impact on production. The indirect effect partially offsets the direct one for oil (so that total response to own futures price is negative), but it outweighs the direct one for meal (leading to a positive total effect).

[25] The rationale for having crushings (production) as an explanatory variable in the regression for purchases (sales) is given in Appendix C.

Table 4.3. Monthly total elasticities of crushings, purchases, and sales with respect to cash and futures prices

Explanatory Variables	Endogenous Variables			
	Soybean Use	Soybean	Output Sales	
	(Crushings)	Purchases	Oil	Meal
Cash prices: soybeans	-0.36	-2.20	-0.25	-0.35
oil			0.50	
meal				0.12
Futures prices: soybeans		0.63		
oil	0.13	0.58	-0.41	0.13
meal	0.23	0.98	0.16	0.10

Long-term equilibrium elasticities are reported in Table 4.4.[26] According to these, in the long term the major adjustment mechanism for processors are stocks rather than crushings (which in long-term equilibrium are identical to purchases and sales). This difference in adjustment patterns is even larger when one examines long-term responses to futures prices. The long-term elasticity of crushings with respect to futures is either zero or virtually zero, while the elasticity with respect to cash prices ranges between 0.53 (meal) and -1.04 (soybeans). This explains why econometric models that use observation horizons longer than one month include cash prices but not futures in the set of variables explaining amounts processed. The main long-term impact of futures is on stocks, which are the endogenous variables with the worst fit in most econometric systems.

The results presented above indicate that cash prices are important to explain crushings, not because firms ignore futures markets (as is implicitly or explicitly assumed in this literature) but because in the long term futures markets mainly influence inventory levels. Models that use cash prices when futures quotes are available may be correct in a reduced-form sense, but these models will inevitably do a poor job of explaining inventory levels. If one assumes that firms ignore futures prices in output decisions, then it is difficult to motivate the use of futures prices in inventory decisions.

Estimation and Derivation of the Empirical Results

The behavioral hypotheses derived in the section dealing with the theoretical model are applicable in the context of the firm's decision horizon. For soybean processors this may be roughly estimated as one week (Tzang and Leuthold 1990). The observation horizon we

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[26]Note that we talk of long *term* and not of long *run*, because in the analysis we consider crushing capacity as an exogenous variable.

Table 4.4. Long-term equilibrium elasticities of endogenous variables with respect to cash and futures prices

Explanatory Variables	Endogenous Variables			
	Crushings (Sales, and Purchases)	Soybean Stocks	Oil Stocks	Meal Stocks
Cash prices: soybeans	-1.04[a]	-8.80	-2.72	-1.04
oil	0.32	0.00	-3.53	0.32
meal	0.53	1.09	1.40	-3.01
Futures prices: soybeans		2.62		
oil	0.06	2.30	4.54	0.06
meal	0.12	2.79	0.31	3.67

[a]The derivation of these elasticities is explained in Appendix C.

employed in empirical analysis, however, is one month. We did so because data on receipts, crushings, and shipments are not available covering periods shorter than one month. On the other hand, we did not use quarterly data because the dynamics of the firm's decisions becomes more difficult to analyze as the observation horizon lengthens. Averages of cash and futures prices tend to converge to each other as the observation horizon lengthens, and the same is true of purchases, crushings, and (weighted) sales. Our hypothesis is that this convergence hides much useful information on firm behavior.

The fact that the observation horizon is longer than the decision horizon poses a problem. For example, whenever the observation horizon exceeds the decision horizon we must include use of material input (Q_t^S) and production (Q_t) as explanatory variables in the regressions for material input purchases (S_t) and output sales (P_t), respectively.[27] But this prevents us from using ordinary least squares to estimate the regressions for S_t and P_t because Q_t^S and Q_t are endogenous. Consequently, we do the estimation by means of a simultaneous equations model.

We also include industry crushing capacity (CAP_t) as an explanatory variable in the regression for crushings. Crushing capacity is expected to be positively related to crushings because it limits the amount of soybeans firms are able to process, and it also captures a time trend.

In expressions (4.10)-(4.12) prices always appear as margins: $(f_{t,t+1} - r_t\, p_t)$, $(f_{t,t+1}/\Phi - r_t\, s_t)$, and $(f^S_{t;t+1} - r_t\, s_t)$. In the empirical test we directly impose these restrictions on prices to avoid multicollinearity, but we use price ratios instead of price differences: $f_{t,t+1}/(r_t\, p_t)$, $(f_{t,t+1}/\Phi)/(r_t\, s_t)$, and $f^S_{t;t+1}/(r_t\, s_t)$. We use ratios for three main reasons. First, they are easy to interpret: the ratios are simply discounted end-of-period rates of return per unit

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[27]See Appendix C for an explanation of this assertion.

of material input. In general, the ratios will be around unity, with values higher (lower) than unity suggesting profits (losses). Second, with the ratio specification we do not need to choose a price index to express the price series in real terms because cash prices are obvious deflators.[28] Third, the problem of not having delivery positions for all months in the futures market is easier to overcome, as discussed below.

The delivery months for soybean oil and meal in the CBOT are January, March, May, July, August, September, October, and December. Hence, in many months we must use $f_{t,t+k}$ $(k > 1)$ instead of $f_{t,t+1}$, because $f_{t,t+1}$ does not exist.[29] But this implies that the ratios for different months are not comparable. For example, the ratio $(f_{t,t+k}/\Phi)/s_t$, which involves a return over $k > 1$ months, cannot be compared with the ratio $(f_{t,t+1}/\Phi)/s_t$, which involves a one-month return only. This suggests converting them to the same base. We chose an annual base for convenience of interpretation of results. Then, the corresponding annualized end-of-period rates of return are $[(f_{t,t+k}/\Phi)/s_t]^{12/k}$, where k is the number of months between the placement of the hedge and the delivery month. This procedure is important because in practice the positions most used for hedging not always are the "nearest" ones. For example, in February most hedges are placed against the May position instead of the March position, therefore the relevant futures price for our purposes is not $f_{Feb,Mar}$ but $f_{Feb,May}$.

Soybean processing involves one material input and not one but two outputs in fixed proportions: oil and meal. Hence, we had to modify the ratio $[(f_{t,t+k}/\Phi)/s_t]^{12/k}$ to make it suitable to analyze the soybean complex. The ratio used is $[(f^{o}_{t;t+k}/\Phi^{o} + f^{m}_{t;t+k}/\Phi^{m})/s_t]^{12/k}$, where superscripts "o" and "m" stand for oil and meal, respectively. This expression should be interpreted in the same way as for the single-output case, with the difference that its

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[28]For example, soybeans have accounted for more than 90 percent of the cost of producing oil and meal.

[29]Examples of nonexistent $f_{t,t+1}$ are $f_{Jan,Feb}$, $f_{Mar,Apr}$, $f_{May,Jun}$, and $f_{Oct,Nov}$.

numerator consists of a composite index of two futures prices of final goods, each one weighted by its corresponding production share.

Following the preceding discussion, the regressions for the soybean complex are

(4.13) $Q_t^s = Q_t^s[\mathrm{RETURN}_t^c, I_t^o, I_t^m, \mathrm{CAP}_t, \mathrm{lag}(Q_t^s)]$ Soybean Crushings

(4.14) $S_t = S_t[\mathrm{RETURN}_t^s, \mathrm{RETURN}_t^c, I_t^s, I_t^o, I_t^m, Q_t^s, \mathrm{lag}(S_t)]$ Soybean Purchases

(4.15) $P_t^o = P_t^o[\mathrm{RETURN}_t^o, I_t^o, Q_t^o, \mathrm{lag}(P_t^o)]$ Oil Sales

(4.16) $P_t^m = P_t^m[\mathrm{RETURN}_t^m, I_t^m, Q_t^m, \mathrm{lag}(P_t^m)]$ Meal Sales

which are to be estimated as a system subject to accounting identities and fixed input-output restrictions.[30] The variables RETURN_t^c, RETURN_t^s, RETURN_t^o and RETURN_t^m are returns per unit of input corresponding to crushings, soybeans, oil, and meal, respectively.[31] In particular, coefficients for RETURNs are expected to be significantly different from zero and positively related to soybean crushings and purchases but negatively related to oil and meal sales.

The data cover September 1965 through December 1986. The period analyzed ends in 1986 because in recent years the processing sector suffered a profound concentration, raising doubts regarding its competitive performance (see Consultants International Group et al. 1986, Bertrand 1988). All prices and quantities for the soybean complex are expressed in dollars per

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[30]See Appendix C for the precise specification of the identities and restrictions.

[31]The actual expressions for RETURN variables are given in Appendix C.

short ton and millions of short tons, respectively. Cash prices are quotations FOB Decatur published by the USDA, and data on crushings, receipts, and shipments are those reported by the U.S. Bureau of the Census.[32] Data sources for crushing capacity are USDA's Fats and Oils--Outlook and Situation, Consultants International Group et al., and the Statistical Annual of the CBOT for the most recent years. These sources only report crushing capacity at the beginning of October, hence capacity for the remaining months was approximated by linear interpolation. Interest rate is the prime rate reported by the USDC's Survey of Current Business. Finally, futures prices employed in the regressions are the average of the highest and lowest futures prices in each month for the selected delivery positions from the Statistical Annual of the CBOT.

The fixed input-output coefficients estimated from the monthly data are $\Phi^o = 5.537$ and $\Phi^m = 1.263$. The coefficients of variation for Φ^o and Φ^m are only 2.35 percent and 0.85 percent, respectively, lending strong support to the assumption that soybean processing is characterized by a Leontief production function. Using these empirical input-output coefficients, we estimated the system of equations (4.13)-(4.16) by means of full information maximum likelihood.[33] We fitted linear and logarithmic specifications of the system and obtained very similar results, particularly regarding the explanatory power of RETURNs. To save space, we report only results of the logarithmic form (see Table 4.5). We preferred this over the linear specification because it had a slightly better fit, and in addition the coefficients are the respective elasticities, which facilitates the interpretation of the results. The goodness-

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[32]Note that available data correspond to receipts and shipments instead of actual purchases and sales, so that we assume that receipts and shipments are identical to purchases and sales, respectively.

[33]We employed full information maximum likelihood because some disturbances exhibit significant autocorrelation, thus rendering incorrect the use of instrumental variables (Johnston, 1984, p. 366).

Table 4.5. Estimated system of equations for U.S. soybean processors employing RETURN variables, 1965:9-1986:12

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Intercept	-0.261	-0.12	-0.106	0.062
	(-5.02)[a]**	(-1.09)	(-3.12)**	(6.10)**
ln(RETURN): crushings	0.075	0.27	*ad hoc*	*ad hoc*
	(3.10)**	(2.35)*		
soybeans	*ad hoc*	0.13	*ad hoc*	*ad hoc*
		(1.15)		
oil	*ad hoc*	*ad hoc*	-0.105	*ad hoc*
			(-4.37)**	
meal	*ad hoc*	*ad hoc*	*ad hoc*	-0.0247
				(-3.74)**
ln(beg. stocks): soybeans	*ad hoc*	-0.241	*ad hoc*	*ad hoc*
		(-5.71)**		
oil	-0.0256	0.015	0.115	*ad hoc*
	(-2.92)**	(0.66)	(5.77)**	
meal	-0.052	-0.044	*ad hoc*	0.0333
	(-4.72)**	(-1.18)		(5.43)**

[a]t statistics are shown in parenthesis.

*Significant at 5%.

**Significant at 1%:

Table 4.5. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
ln(crushings)		0.78		
		(7.13)**		
ln(production): oil			0.697	
			(16.43)**	
meal				0.9665
				(100.34)**
ln(crushing capacity)	0.291			
	(7.80)**			
ln(lagged endog.): lag 1	0.874	0.422		
	(18.94)**	(8.27)**		
lag 2	-0.102			
	(-2.40)*			
Dummy: February	-0.080			
	(-4.70)**			
March	0.072			
	(4.53)**			
April	-0.047			
	(-2.98)**			

Table 4.5. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Dummy: May	0.039			
	(2.56)*			
June	-0.062			
	(-4.28)**			
September	-0.069			
	(-5.86)**			
October	0.178	0.59		
	(9.96)**	(5.63)**		
December	0.047			
	(3.06)**			
AUTOC. COEFF.: t-1			0.362	-0.225
			(4.50)**	(-3.31)**
t-2			0.280	
			(4.74)**	
t-3				0.331
				(4.43)**
t-12		0.446		0.174
		(7.34)**		(2.48)*

Table 4.5. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
STATISTICS:				
R^2	0.962	0.933	0.958	0.990
Mean value of dep. variable	0.745	0.668	-0.967	0.512
Std. error of regression	0.0509	0.147	0.0536	0.0235
SYSTEM STATISTICS:				
$R_p^2 = 0.998$	Log Likelihood Function = 1214.31		Observations = 242	

of-fit statistic for the logarithmic system is $R_p^2 = 0.998$.[34] The R^2s for the individual equations range from 0.933 to 0.990.[35]

We employed the coefficients reported in Table 4.5 to obtain Tables 4.2 and 4.3, by taking an average of $\Phi^o = 5.537$, $\Phi^m = 1.263$, $(f^o_{t;t+k}/\Phi^o)/(f^o_{t;t+k}/\Phi^o + f^m_{t;t+k}/\Phi^m) = 0.372$, $(f^m_{t;t+k}/\Phi^m)/(f^o_{t;t+k}/\Phi^o + f^m_{t;t+k}/\Phi^m) = 0.628$, $k = 4.787$, $h = 4.918$, and $r_t = 1.095$. Then, for example, partial elasticity of input purchases with respect to oil futures price is $(0.27\ k\ 0.372) = 0.48$, and total elasticity of oil sales with respect to soybean cash price is $(-0.075\ k\ 0.697) = -0.25$.

We included monthly dummy variables in the regression for soybean crushings to model the seasonal pattern.[36] In the equation for soybean purchases we modeled seasonality by means of a twelve-month correlation structure in the error, plus a dummy variable accounting for October. October marks the beginning of the crushing year, and purchases are abnormally high compared to other months: October accounted for at least 15 percent of annual purchases during the period 1965/66-1985/86, with the only exception of year 1984/85 in which that percentage was 11.8. In the equation for meal sales the seasonal pattern was

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[34]The statistic R_p^2 is the *pseudo* R^2 introduced by Baxter and Cragg (1970). This is defined as

$$R_p^2 = 1 - \exp[2\,(L_\omega - L_\Omega^{max})/N]$$

where L_ω is the maximum of the log likelihood function when only intercepts are used, L_Ω^{max} is the maximum of the log likelihood function when all coefficients are included in the model, and N is the number of observations.

[35]The R^2 for each individual equation was calculated as suggested by Maddala (1988), by taking the squared correlation between predicted and actual endogenous variables.

[36]For example, the dummy variable February equals 1 if t = February, and equals 0 otherwise.

captured by autocorrelation coefficients at lags 3 and 12. Both the equations for oil and meal sales exhibited significant first-order autocorrelation, and the equation for oil sales also had significant second-order autocorrelation.

Crushing capacity is a highly significant explanatory variable of the amount of soybeans processed. The short-run elasticity of crushings with respect to capacity is 0.291. Although this value is apparently low, it yields a long-term equilibrium elasticity of 0.84.[37] The long-term elasticities of soybean and meal stocks with respect to capacity have reasonable magnitudes (0.68 and 0.84, respectively), but that of oil stocks (e.g., 2.19) seems rather high. This is mostly due to the accumulation of oil stocks that occurred at the same time that the processing industry expanded its crushing capacity.[38]

As hypothesized, beginning inventories of final goods have a significantly positive impact on their respective sales, while beginning stocks of soybeans have a significantly negative effect on material input purchases. The corresponding total elasticities, however, are very low: -0.017 for meal sales, 0.098 for oil sales, and -0.241 for soybean purchases (see Table 4.3).[39] In addition, the empirical findings indicate that quantity processed is negatively related to beginning inventories of oil and meal, suggesting that production and output storage are not separated in the U.S. soybean-processing sector. It is this negative effect of beginning meal inventories on crushings that is accountable for the negative total elasticity of meal sales with respect to beginning meal stocks.

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[37]The methodology to obtain long-term elasticities from structural parameters reported in Table 4.5 is explained in Appendix C.

[38]For example, the ratio of oil stocks to monthly crushing capacity averaged 4 percent in 1967:10/1975:9, 7 percent in 1975:10/1980:9, 13 percent in 1980:10/1983:9, and 7 percent in 1983:10/1986:9. In the same periods the average monthly crushing capacities were 2.2, 3.2, 3.7, and 3.5 million short tons, respectively.

[39]Notice that the total elasticities of oil and meal sales with respect to their own beginning stocks include the indirect effect of beginning stocks on oil and meal production.

The most important empirical result regarding our theoretical model is that the RETURN variables had the hypothesized effects and were significant. The equation for soybean purchases is the only one in which RETURNs seem to have little or no explanatory power. This is due to multicollinearity, caused by high correlation between RETURNs for crushings and soybeans. Proof of this is that deleting $RETURN_t^S$ from the equation for soybean purchases yields a coefficient for $RETURN_t^C$ equal to 0.357 with a t statistic of 4.62. Similarly, if we delete instead $RETURN_t^C$ from the same equation, the coefficient for $RETURN_t^S$ becomes 0.294 with a t statistic of 3.96.

The variables indicated as *ad hoc* are those that could be included in an ad hoc model of the sector, but which are not predicted by the theoretical model. The *ad hoc* variables were excluded from the system reported in Table 4.5. To test if the *ad hoc* variables have any explanatory power we ran an unrestricted version of the model which had all these additional variables. The resulting likelihood ratio was 24.14 with 16 degrees of freedom, while the critical $\chi^2_{16;0.05}$ is 26.30. This indicates that the null hypothesis that all of the coefficients pertaining to the *ad hoc* variables are not significantly different from zero could not be rejected. Of these *ad hoc* variables, the only one that was significantly different from zero at the 5 percent level (t = -2.14) was beginning stocks of soybeans in the regression corresponding to oil sales, but its coefficient was negative. Clearly, this relationship should be positive (assuming that oil and soybeans compete for storage resources), or insignificant (as predicted by the theoretical model). Interestingly, had we begun by searching the data for variables that were significant and of the correct sign, we would with two exceptions (oil and meal stocks did not significantly influence soybean purchases) have arrived at the model structure predicted by the theory.

Price Expectations

An interesting question that arises is how well the system of equations estimated by means of the RETURN variables compares against systems employing *expected* prices rather than futures. To answer it we estimated similar regressions, using indexes reflecting *perfect foresight* and *naive price expectations* (PERFORs and NAIVEs, respectively) instead of RETURNs.[40] The results using PERFOR and NAIVE variables are reported in Tables 4.6 and 4.7, respectively.

The statistical significance of the RETURN coefficients is even more evident in Tables 4.6 and 4.7. In the case of the perfect foresight index, the only coefficients significantly different from zero at the 5 percent level are those corresponding to the equations for soybean crushings and purchases. At the 1 percent level of significance only the soybean PERFOR coefficient is different from zero. Moreover, one of the significant coefficients has the wrong sign (i.e., crushing PERFOR in soybean purchase equation). For naive expectations all the coefficients corresponding to NAIVE variables have the correct signs, but none of them is significantly different from zero at the 5 percent level of significance. In addition, the systems estimated by means of the PERFOR and NAIVE variables had more problems of autocorrelation in the residuals. In the case of the PERFOR system, it was necessary to incorporate lagged endogenous variables in the oil and meal regressions. In the PERFOR system we had to include a lagged endogenous variable in the meal regression, and a third-order autocorrelation coefficient in the oil regression. In summary, it is clear that the two models fitted with price expectation indexes have poorer explanatory power than the one using futures prices. This means that soybean processors use futures markets to make their physical decisions.

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[40]The precise definition of the PERFOR and NAIVE variables is given in Appendix C.

Table 4.6. Estimated system of equations for U.S. soybean processors employing PERFOR variables, 1965:9-1986:12

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Intercept	-0.271	-0.19	-0.275	0.046
	(-5.14)**[a]	(-1.77)	(-3.97)**	(4.03)**
ln(PERFOR): crushings	0.0161	-0.063		
	(2.34)*	(-2.10)*		
soybeans		0.092		
		(2.84)**		
oil			-0.0086	
			(-1.31)	
meal				0.0010
				(0.52)
ln(beg. stocks): soybeans		-0.222		
		(-5.31)**		
oil	-0.0298	-0.018	0.136	
	(-3.44)**	(-0.79)	(5.62)**	
meal	-0.048	-0.050		0.0221
	(-4.12)**	(-1.26)		(3.57)**

[a]t statistics are shown in parenthesis.

*Significant at 5%.

**Significant at 1%.

Table 4.6. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
ln(crushings)		0.83		
		(7.47)**		
ln(production): oil			0.662	
			(13.16)**	
meal				0.920
				(43.31)**
ln(crushing capacity)	0.291			
	(7.51)**			
ln(lagged endog.): lag 1	0.912	0.428	-0.208	0.050
	(21.89)**	(8.19)**	(-4.62)**	(2.10)*
lag 2	-0.122			
	(-2.89)**			
Dummy: February	-0.081			
	(-4.76)**			
March	0.077			
	(4.90)**			
April	-0.051			
	(-3.20)**			

Table 4.6. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Dummy: May	0.042			
	(2.96)**			
June	-0.064			
	(-4.11)**			
September	-0.070			
	(-6.19)**			
October	0.198	0.70		
	(12.92)**	(6.71)**		
December	0.047			
	(3.03)**			
AUTOC. COEFF.: t-1			0.668	-0.258
			(10.39)**	(-3.84)**
t-3			0.209	0.326
			(3.60)**	(4.36)**
t-12		0.497		0.161
		(8.76)**		(2.24)*

Table 4.6. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
STATISTICS:				
R^2	0.961	0.930	0.957	0.990
Mean value of dep. variable	0.745	0.668	-0.967	0.512
Std. error of regression	0.0510	0.151	0.0541	0.0239
SYSTEM STATISTICS:				
$R_p^2 = 0.997$	Log Likelihood Function = 1197.33		Observations = 242	

Table 4.7. Estimated system of equations for U.S. soybean processors employing NAIVE variables, 1965:9-1986:12

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Intercept	-0.252	-0.14	-0.290	0.041
	(-4.57)**[a]	(-1.13)	(-4.38)**	(3.50)**
ln(NAIVE): crushings	0.0065	0.035		
	(0.89)	(1.85)		
soybeans		0.60		
		(1.15)		
oil			-0.70	
			(-1.69)	
meal				-0.068
				(-0.97)
ln(beg. stocks): soybeans		-0.225		
		(-5.02)**		
oil	-0.0267	0.004	0.083	
	(-2.88)**	(0.17)	(4.43)**	
meal	-0.047	-0.019		0.0213
	(-3.79)**	(-0.46)		(3.59)**

[a] t statistics are shown in parenthesis.

*Significant at 5%.

**Significant at 1%.

Table 4.7. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
ln(crushings)		0.90		
		(7.59)**		
ln(production): oil			0.605	
			(13.88)**	
meal				0.916
				(43.01)**
ln(crushing capacity)	0.263			
	(6.74)**			
ln(lagged endog.): lag 1	0.950	0.421		0.049
	(22.13)**	(7.60)**		(2.13)*
lag 2	-0.142			
	(-3.42)**			
Dummy: February	-0.080			
	(-4.88)**			
March	0.083			
	(5.11)**			
April	-0.051			
	(-3.16)**			

Table 4.7. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Dummy: May	0.047			
	(3.16)**			
June	-0.061			
	(-4.17)**			
September	-0.068			
	(-5.75)**			
October	0.207	0.68		
	(14.20)**	(6.23)**		
December	0.046			
	(2.78)**			
AUTOC. COEFF.: t-1			0.348	-0.266
			(4.77)**	(-3.95)**
t-2			0.254	
			(4.12)**	
t-3			0.194	0.317
			(3.02)**	(4.23)**
t-12		0.517		0.160
		(9.29)**		(2.20)*

Table 4.7. continued

Explanatory Variables	Endogenous Variables			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
STATISTICS:				
R^2	0.960	0.929	0.957	0.990
Mean value of dep. variable	0.745	0.668	-0.967	0.512
Std. error of regression	0.0519	0.151	0.0547	0.0238
SYSTEM STATISTICS:				
$R_p^2 = 0.997$	Log Likelihood Function = 1189.56		Observations = 242	

It is important to note that the NAIVE variables represent *cash-cash marketing margins*, which are the central paradigm in the analysis of marketing industries (Lence, Hayes, and Meyers 1992).[41] Our results support the conclusions by Lence, Hayes, and Meyers, who state that "futures prices and their relationship with cash prices seem to be important factors affecting processing decisions." In addition, our findings provide evidence that using cash-cash marketing margins to study market performance and economic behavior of marketing firms in the presence of futures markets is inappropriate.

Conclusions

The analysis provides evidence that in the short run purchases and processing of material input and production and sales of final product should, and do, respond to different explanatory variables and in different ways than is commonly accepted. We achieve these results by introducing the realistic assumption that firms make production, purchasing, and selling decisions in order to take advantage of cash and forward price differentials, while hedging the inherent risk in forward markets. Although the focus of our study is on short-run (monthly) behavior, the results show that inferences made about long-term firm behavior derived by aggregating over short-term decisions are different in some respects from the inferences one would draw from medium- or long-run models.

The model presented is capable of identifying the individual effect of each cash and forward price on purchases, processing, and sales. In the particular industry studied (U.S. soybean processing), we found that the cash price of the material input (soybeans) is in general the single most important price affecting processors' decisions. We also found that even

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[41] Cash-cash marketing margins are the relationship between cash prices of final product and material input measured at the same point in time.

though in the short term material input use (i.e., soybean crushings) is not affected by cash prices of output (i.e., oil and meal), in the long term crushings are almost as responsive to these as to soybean cash prices. In contrast, the impact of the output futures price on processing is noticeable in the short term, but it is almost negligible in the long term. These results may help explain why studies employing annual data generally use cash prices but not futures to model the supply of processing services. The major effect of futures prices is on purchases and sales in the short term and on stocks in the long term. The results also show that soybean processors respond more logically to futures prices than they do to prices formed by naive expectations or even perfect foresight.

CHAPTER V. SUMMARY AND CONCLUSIONS

In this study we analyze the behavior of a forward-looking expected-utility-maximizing competitive firm and examine how it compares with an otherwise identical myopic firm.

We first address the case of a speculative storing firm in the absence of forward markets. This model characterizes speculative firms storing a commodity, and speculative holders of stocks, bonds, and other nontransformable assets. In this scenario, changes in beginning inventories, in current price, or in the interest rate cause wealth changes and consequently modify the firm's risk behavior unless the utility function is CARA. Wealth fluctuations do not alter risk behavior if the firm is CARA for both forward-looking and myopic attitudes; in this instance we can show that storage is negatively associated with current cash price and the interest rate, and that it is independent from the level of beginning stocks. In contrast, if the firm is DARA or IARA, the storage response to the mentioned variables is generally ambiguous.

By restricting the firm's utility to be CARA, we can show that the forward-looking reservation price is higher than the risk-neutral one if (a) the next-date price is independently distributed from all posterior prices, or (b) the decision maker is sufficiently absolute risk-averse, or (c) the price follows a stationary autoregressive process and the agent is sufficiently forward looking. Therefore, the forward-looking CARA firm is willing to hold inventories at levels where the discounted expected next-date price is less than the current price plus marginal storage cost. It follows that the forward-looking CARA firm will store more than the risk-neutral one at sufficiently low storage levels. These results are in sharp contrast to myopic risk-averse behavior. We show, however, that at sufficiently high storage levels risk-neutral storage exceeds forward-looking CARA storage. These findings may explain the ambiguous response of forward-looking CARA storage to changes in the next-date expected price, in the next-date mean-preserving price spread, or in the degree of absolute risk aversion.

The behavioral differences between myopic risk-averse and forward-looking CARA firms are attributable to the fact that the first cares only about revenue risk, whereas the second is concerned about both revenue and input cost risks. The myopic firm acts as if it intends to exit the market at the current cycle's end, and therefore dismisses the possibility of buying product to store in the future. For such a firm, the only risk effect of storage is adding revenue risk. In contrast, the forward-looking firm plans to stay in the market after the current storage cycle, so it takes into account the possibility that at the next decision date it may be optimal to buy product to store. Because of this, for the forward-looking firm current storage not only increases revenue risk but also lowers cost risk. This means that the forward-looking firm may be willing to hold inventories even if the one-period expected return from storage is negative.

The forward-looking CARA model of speculative storage explains real-world observations that are not compatible with myopic risk-averse behavior. For example, firms practice sequential marketing, hold output and/or input reserves, and spread transactions over time.

When the speculative storing firm is allowed to trade forward, its behavior changes substantially. The firm separates storage decisions from hedging decisions. Storage is independent from the subjective joint distribution of random variables and from the decision maker's degree of risk aversion. If positive, optimal storage is such that the discounted current forward price equals the current cash price plus the marginal storage cost, independent of forward-looking or myopic behavior. Forward-looking and myopic attitudes, however, are reflected in the optimal hedging decisions. The optimal hedge for a myopic risk-averse firm that perceives the forward price to be unbiased is the full hedge; i.e., selling forward the entire quantity stored. In contrast, the optimal forward-looking hedge under an unbiased forward price generally will differ from the full hedge and will depend on the firm's degree of risk aversion. Moreover, if the forward-looking firm is CARA and (a) the next-date cash price is independently distributed from the next-date forward price and from all posterior (cash and forward) prices, or (b) the agent is sufficiently absolute risk-averse, then its optimal hedge under an unbiased forward price will be smaller than the full hedge.

The optimal forward-looking CARA hedge under an unbiased forward price is smaller than the full hedge because the forward-looking firm assigns a positive probability that it will store at the next trading date. Next-date storage is negatively correlated with next-date cash price, and therefore next-date storage eliminates part of the risk of next-date cash price. Hence, next-date storage is a substitute for current hedging, thus reducing the size of the hedge required to minimize the risk of next-date cash price. These results regarding the optimal forward-looking hedge offer an alternative explanation of the stylized fact that farmers do hedge their entire grain stocks. Our findings are also relevant for empirical research concerned with the empirical estimation of optimal hedges in the presence of futures markets rather than forward markets.

By restricting the firm's production function to be nonstochastic Leontief, the speculative storage scenario can be readily modified to model a productive nonstoring firm. If output and material input cash prices are positively related, most of the findings for the speculative storage framework follow by noting that the firm's activity is "production" rather than "storage." Also, the rationale for the results derived for the productive nonstoring firm is completely analogous to that for speculative storage.

Among the main results from the productive nonstoring scenario is that the forward-looking CARA firm will produce more than the risk-neutral one at sufficiently low output levels, and less at sufficiently high levels of production. This finding has important implications. For example, the firm's production response to risk has been assumed traditionally to be the same at all output levels. This may bias empirical results towards rejection of the hypothesis that risk affects the behavior of the firm. Also, relaxing the myopic constraint seems particularly relevant for studies simultaneously involving rational expectations and risk aversion.

In the final analysis we present the most general model, in which firms are allowed to produce and to store and trade forward both output and material input. The most important result from this setting is that "physical" decisions are separated from hedging decisions; i.e., separation holds regardless of forward-looking or myopic behavior. Physical decisions are independent from the firm's degree of risk aversion and from the subjective joint distribution of random variables.

All physical decisions are shown to depend only on current forward prices and cash prices, the interest rate, and the storage and production cost functions. Because of this and the independence from hedging, the main behavioral hypotheses regarding physical decisions are readily testable empirically. We employed monthly data from the U.S. soybean-processing industry to test the theoretical model advanced, with the results strongly supporting the model. The results also show that soybean processors respond more logically to futures prices than they do to prices formed by naive expectations or even perfect foresight. The empirical results are particularly important because our model allows us to better understand the short-term behavior of the firm and to derive meaningful long-term response parameters from short-term parameter estimates.

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APPENDIX A. APPENDIX TO CHAPTER II

Derivation of FOC (2.7)

The FOCs corresponding to the Lagrangian for $t < T$ are

$$\text{(A1)}\quad \frac{\partial \pounds_t}{\partial P_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} \; \frac{\partial I_{t+1}}{\partial P_t}\right]$$

$$+ r_t \, r_{t+1} \cdots r_{T-2} \, r_{T-1} \, [p_t + i'(\cdot)] \, M_t'(W_T, e_t) - \eta_t = 0$$

plus (2.8). But note that

$$\text{(A2)}\quad \frac{\partial I_{t+1}}{\partial P_t} = -1$$

$$\text{(A3)}\quad \frac{\partial I_{t+1}}{\partial I_t} = 1$$

Also,

$$\text{(A4)}\quad \frac{\partial M_t(W_T, e_t)}{\partial I_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} \; \frac{\partial I_{t+1}}{\partial I_t}\right]$$

$$- r_t \, r_{t+1} \cdots r_{T-2} \, r_{T-1} \, i'(\cdot) \, M_t'(W_T, e_t) + \eta_t$$

$$= r_t \, r_{t+1} \cdots r_{T-2} \, r_{T-1} \, p_t \, M_t'(W_T, e_t)$$

where the second equality in (A4) is obtained by using expressions (A1) through (A3). It follows from (A4) that

(A5) $$\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} = r_{t+1}\, r_{t+2} \cdots r_{T-2}\, r_{T-1}\, p_{t+1}\, M_{t+1}'(W_T, e_{t+1})$$

Substitution of (A2) and (A5) into FOC (A1), and rearrangement, yields expression (2.7).

Derivation of FOC (2.9)

For the risk-neutral firm, the dynamic programming algorithm is analogous to (2.3) but with W_T instead of $U(W_T)$. Therefore, the FOCs corresponding to the terminal date T are

(A6) $$\frac{\partial \pounds_T}{\partial P_T} = (p_T + i') - \eta_T = 0$$

(A7) $$\frac{\partial \pounds_T}{\partial \eta_T} = I_T - P_T = 0,\ \eta_T > 0,\ \eta_T \frac{\partial \pounds_T}{\partial \eta_T} = 0$$

and optimal terminal sales are $P_T = I_T$. For any period $t < T$, FOCs are as follows:

(A8) $$\frac{\partial \pounds_t}{\partial P_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial P_t}\right] + r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, [p_t + i'(\cdot)] - \eta_t = 0$$

plus (2.10). But (A2) and (A3) still apply, and the expressions analogous to (A4) and (A5) are, respectively,

(A9) $$\frac{\partial M_t(W_T, e_t)}{\partial I_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial I_t}\right] - r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, i'(\cdot) + \eta_t$$

$$= r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, p_t$$

(A10) $$\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} = r_{t+1}\, r_{t+2} \cdots r_{T-2}\, r_{T-1}\, p_{t+1}$$

Expression (2.9) is obtained by replacing (A2) and (A10) into FOC (A8).

Proof of Propositions 2.1 and 2.2

The Lagrangian multiplier vanishes ($\eta_0 = 0$) if storage is positive ($I_1 = I_0 - P_0 > 0$). Comparative statics can be obtained by totally differentiating FOC (2.7) as expressed in (A11)[42]

$$\text{(A11)}\quad £_P = \mathbb{E}_0\{[r_0\,(p_0 + i'(I_0 - P_0)) - p_1]\, M_1'(W_T, e_1)\} = 0$$

and then calculating the effect of any variable x on sales as $\dfrac{\partial P_0}{\partial x} = -\dfrac{£_{Px}}{£_{PP}}$. The derivatives of $£_P$ are:[43]

$$\text{(A12)}\quad £_{PP} = r_1 \ldots r_{T-1}\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]^2\, M_1''\} - r_0\, i''\, M_0' < 0$$

$$\text{(A13)}\quad £_{Pp} = r_0 \ldots r_{T-1}\, P_0\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} + r_0\, M_0' \gtrless 0$$

$$\text{(A14)}\quad £_{Pr} = r_1 \ldots r_{T-1}\, (p_0\, P_0 - i)\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} + (p_0 + i')\, M_0' \gtrless 0$$

$$\text{(A15)}\quad £_{PI} = r_0 \ldots r_{T-1}\, p_0\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} - £_{PP} \gtrless 0$$

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[42]To make notation less cumbersome, in this section we use $£_P$ to denote $\partial £_t / \partial P_t$. The meaning of the remaining derivatives should be clear from the context.

[43]To simplify notation, whenever we refer to $£_{P\mu}$ and $£_{P\sigma}$ we assume they are evaluated at $\sigma_{0,1} = 1$.

(A16) $\pounds_{P\mu} = r_1 \ldots r_{T-1}\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, P_1\, M_1''\} - M_0' \gtrless 0$

(A17) $\pounds_{P\sigma} = r_1 \ldots r_{T-1}\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, P_1\, (p_1 - \mu_{0,1})\, M_1''\}$

$$- \mathbb{E}_0\{(p_1 - \mu_{0,1})\, M_1'\} \gtrless 0$$

The sign of $\mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\}$ is ambiguous in general, but for CARA or myopic firms it can be inferred.

For CARA firms we have

(A18) $\mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} = -\lambda\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, M_1'\} = 0$

by FOC. Hence, $\pounds_{Pp} = r_0\, M_0' > 0$, $\pounds_{Pr} = (p_0 + i')\, M_0' > 0$, and $\pounds_{PI} = -\pounds_{PP} > 0$. Also, from $\pounds_{Pp} > 0$ it follows that $\partial P_1/\partial p_1 \geq 0$ for CARA. Then:

(A19) $\mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, P_1\, M_1''\} = \lambda\, \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, (P_k - P_1)\, M_1'\} \geq 0$

where P_k is a constant equal to P_1 when p_1 equals $[r_0\,(p_0 + i')]$. Expression (A19) is nonnegative because $M_1' > 0$, and $(P_k - P_1)$ is positive (negative) whenever $[r_0\,(p_0 + i') - p_1]$ is positive (negative). Therefore, $\pounds_{P\mu} \gtrless 0$ even for the CARA firm. The sign of $\pounds_{P\sigma}$ is also ambiguous in general; for example, $\mathbb{E}_0[(p_1 - \mu_{0,1})\, M_1'] = \mathrm{Cov}(p_1, M_1')$ may be positive or negative for CARA forward-looking firms, as inferred from the proof of Proposition 2.4.

For myopic firms, we have date 0 = T-1 and $P_1 = I_T = I_1$. Therefore,

(A20) $\mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} = \mathbb{E}_0\{[r_0\,(p_0 + i') - p_1]\, (\lambda_k - \lambda)\, M_1'\} \begin{cases} < 0 \text{ if DARA} \\ = 0 \text{ if CARA} \\ > 0 \text{ if IARA} \end{cases}$

$$\text{(A21)}\quad \mathbb{E}_0\{[r_0(p_0+i')-p_1]P_1M_1''\} = I_1\,\mathbb{E}_0\{[r_0(p_0+i')-p_1]M_1''\}\begin{cases}<0 \text{ if DARA}\\ =0 \text{ if CARA}\\ >0 \text{ if IARA}\end{cases}$$

$$\text{(A22)}\quad \mathbb{E}_0\{[r_0(p_0+i')-p_1]P_1(p_1-\mu_{0,1})M_1''\} = -I_1\,\mathbb{E}_0\{[r_0(p_0+i')-p_1]^2M_1''\}$$

$$-I_1[\mu_{0,1}-r_0(p_0+i')]\,\mathbb{E}_0\{[r_0(p_0+i')-p_1]M_1''\}\begin{cases}>0 \text{ if DARA or CARA}\\ \gtrless 0 \text{ if IARA}\end{cases}$$

where λ_k is a constant equal to λ when p_1 equals $[r_0(p_0+i')]$. The sign of (A20) follows because $M_1' > 0$, and for DARA $(\lambda_k-\lambda)$ is positive if $[r_0(p_0+i')-p_1]$ is negative, and vice versa. For IARA we have $(\lambda_k-\lambda)$ positive (negative) when $[r_0(p_0+i')-p_1]$ is positive (negative). Therefore, for a myopic firm we have

$$\text{(A23)}\quad \pounds_{Pp}\begin{cases}>0 \text{ if DARA and } P_0 \le 0\text{; or if IARA and } P_0 \ge 0\\ \gtrless 0 \text{ if DARA and } P_0 > 0\text{; or if IARA and } P_0 < 0\end{cases}$$

$$\text{(A24)}\quad \pounds_{Pr}\begin{cases}>0 \text{ if DARA and } p_0P_0 \le i\text{; or if IARA and } p_0P_0 \ge i\\ \gtrless 0 \text{ if DARA and } p_0P_0 > i\text{; or if IARA and } p_0P_0 < i\end{cases}$$

$$\text{(A25)}\quad \pounds_{PI}\begin{cases}< -\pounds_{PP} \text{ if DARA}\\ > -\pounds_{PP} > 0 \text{ if IARA}\end{cases}$$

$$\text{(A26)}\quad \pounds_{P\mu}\begin{cases}<0 \text{ if DARA or CARA}\\ \gtrless 0 \text{ if IARA}\end{cases}$$

$$\text{(A27)}\quad \pounds_{P\sigma}\begin{cases}>0 \text{ if DARA or CARA}\\ \gtrless 0 \text{ if IARA}\end{cases}$$

To show the effect of the degree of absolute risk aversion on the myopic firm, take two firms A and B such that $\lambda_A(W_T) > \lambda_B(W_T)$ for all W_T.[44] Rewrite FOC corresponding to firm A as:

$$\text{(A28)} \quad \int_0^{r_0(p_0+i')} \frac{M_{1A}'(W_T)}{M_{1A}'(W_k)} [r_0(p_0+i') - p_1]\, p_1(p_1)\, dp_1 + \int_{r_0(p_0+i')}^{\infty} \frac{M_{1A}'(W_T)}{M_{1A}'(W_k)} [r_0(p_0+i') - p_1]\, p_1(p_1)\, dp_1 = 0$$

where W_k is terminal wealth corresponding to $p_1 = r_0(p_0 + i')$. Equality (A28) is satisfied at firm A's optimum sales level (P_{0A}). For firm B, a similar expression to (A28) evaluated at P_{0A} is

$$\text{(A29)} \quad \int_0^{r_0(p_0+i')} \frac{M_{1B}'(W_T)}{M_{1B}'(W_k)} [r_0(p_0+i') - p_1]\, p_1(p_1)\, dp_1 + \int_{r_0(p_0+i')}^{\infty} \frac{M_{1B}'(W_T)}{M_{1B}'(W_k)} [r_0(p_0+i') - p_1]\, p_1(p_1)\, dp_1 < 0$$

Negativity of expression (A29) can be proven as follows. Subtract (A28) from (A29) to get

$$\text{(A30)} \quad \int_0^{r_0(p_0+i')} \left[\frac{M_{1B}'(W_T)}{M_{1B}'(W_k)} - \frac{M_{1A}'(W_T)}{M_{1A}'(W_k)}\right] [r_0(p_0+i') - p_1]\, p_1(p_1)\, dp_1 + \int_{r_0(p_0+i')}^{\infty} \left[\frac{M_{1B}'(W_T)}{M_{1B}'(W_k)} - \frac{M_{1A}'(W_T)}{M_{1A}'(W_k)}\right] [r_0(p_0+i') - p_1]\, p_1(p_1)\, dp_1$$

Terminal wealth is higher the higher next-date price because the firm is myopic. Hence,

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44This proof follows Holthausen's methodology.

$W_k > W_T$ in the first integral, and $W_k < W_T$ in the second integral. Applying inequality (20) in Pratt (1964), it follows that the term involving ratios is negative in the first integral, and positive in the second integral. Also, the term $[r_0 (p_0 + i') - p_1]$ is positive in the first integral, and negative in the second. Therefore, both integrals are negative, (A30) is negative, and (A29) must be negative. We conclude that firm B's optimum sales (P_{0B}) must be lower than P_{0A} because firm B's FOC is negative when evaluated at P_{0A}. Hence, for myopic firms we have $P_{0A} > P_{0B}$ if $\lambda_A > \lambda_B$.

The degree of absolute risk aversion has an ambiguous effect on forward-looking sales, even for CARA firms. This can be inferred from the preceding paragraph because $W_k \gtrless W_T$ in either integral of (A30) if firms are forward-looking. Therefore, (A30) has an ambiguous sign, and forward-looking sales may increase or decrease with the degree of absolute risk aversion.

Comparative statics for storage follow by applying the identity $I_{t+1} = I_t - P_t$.

Example of Optimum Myopic CARA Storage

If the firm is CARA, then

$$\text{(A31)}\quad U(W_T) = -\exp[-\lambda\,(r_{-1}\, r_0\, r_1 \cdots r_{T-1}\, W_{-1} + r_0\, r_1 \cdots r_{T-1}\, \pi_0 + r_1 \cdots r_{T-1}\, \pi_1 + \dots + r_{T-1}\, \pi_{T-1} + \pi_T)]$$

$$= \exp(-\lambda\, r_{-1}\, r_0\, r_1 \cdots r_{T-1}\, W_{-1}) \exp(-\lambda\, r_0\, r_1 \cdots r_{T-1}\, \pi_0)$$

$$\exp(-\lambda\, r_1 \cdots r_{T-1}\, \pi_1) \dots \exp(-\lambda\, r_{T-1}\, \pi_{T-1}) [-\exp(-\lambda\, \pi_T)]$$

where exp(x) denotes the base of natural logarithms raised to the power x. The optimization problem may be expressed as[45]

$$\text{(A32)}\quad \max_{P_0 \le I_0} \mathbb{E}_0[U(W_T)] = \exp(-\lambda\, r_{-1}\, r_0\, r_1 \ldots r_{T-1}\, W_{-1})$$

$$\max_{P_0 \le I_0}\{\exp(-\lambda\, r_0\, r_1 \ldots r_{T-1}\, \pi_0)\, \mathbb{E}_0[\max_{P_1 \le I_1} \exp(-\lambda\, r_1 \ldots r_{T-1}\, \pi_1)$$

$$\ldots \max_{P_{T-1} \le I_{T-1}}(\exp(-\lambda\, r_{T-1}\, \pi_{T-1})\, \mathbb{E}_{T-1}(\max_{P_T \le I_T}(-\exp(-\lambda\, \pi_T)))))]\}$$

But the first term in the right-hand side of (A32) does not affect the solution, and $I_{t+1} = I_{t-1} - P_{t-1}$. We also know that $P_T = I_T$ is optimal [see FOCs (2.4) and (2.5)]. Therefore, an alternative specification of (A32) is

$$\text{(A33)}\quad \max_{I_1 \ge 0} \mathbb{E}_0[U(W_T)] = \max_{I_1 \ge 0}\{\exp(-\lambda\, r_0\, r_1 \ldots r_{T-1}\, \pi_0)$$

$$\mathbb{E}_0[\max_{I_2 \ge 0}(\exp(-\lambda\, r_1 \ldots r_{T-1}\, \pi_1)$$

$$\ldots \max_{I_T \ge 0}(\exp(-\lambda\, r_{T-1}\, \pi_{T-1})\, \mathbb{E}_{T-1}(-\exp(-\lambda\, p_T\, I_T)))))]\}$$

Optimum beginning inventories at date T are obtained by solving the corresponding Kuhn-Tucker conditions for I_T, i.e.,

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[45]See Bertsekas (1976).

$$\text{(A34)} \quad \frac{\partial \pounds_{T-1}}{\partial I_T} = -\lambda \exp(-\lambda\, r_{T-1}\, \pi_{T-1})\, \mathbb{E}_{T-1}\{[p_T - r_{T-1}\,(p_{T-1} + i')]\,[-\exp(-\lambda\, p_T\, I_T)]\} \leq 0,$$

$$I_T \geq 0,\ I_T\, \frac{\partial \pounds_{T-1}}{\partial I_T} = 0$$

where $\pounds_{T-1} = \exp(-\lambda\, r_{T-1}\, \pi_{T-1})\, \mathbb{E}_{T-1}[-\exp(-\lambda\, p_T\, I_T)]$. Under the adopted assumptions, we have

$$\text{(A35)} \quad \mathbb{E}_{T-1}\{[p_T - r_{T-1}\,(p_{T-1} + i')]\,[-\exp(-\lambda\, p_T\, I_T)]\} = \int_{p_T} [p_T - r_{T-1}\,(p_{T-1} + 2\,\Theta\, I_T)]$$

$$[-\exp(-\lambda\, p_T\, I_T)]\; dn(p_T)$$

where $dn(p_T) = \dfrac{1}{\sigma_T \sqrt{2\,\pi}} \exp\left[-\frac{1}{2}\left(\dfrac{p_T - \mu_T}{\sigma_T}\right)^2\right] dp_T$

But

$$\text{(A36)} \quad -\exp(-\lambda\, p_T\, I_T)\, \exp\left[-\frac{1}{2}\left(\frac{p_T - \mu_T}{\sigma_T}\right)^2\right] = -\exp[(\lambda\, \sigma_T\, I_T)^2/2]\, \exp(-\lambda\, \mu_T\, I_T)$$

$$\exp\left\{-\frac{1}{2}\left[\frac{p_T - (\mu_T - \lambda\, \sigma_T^2\, I_T)}{\sigma_T}\right]^2\right\}$$

The first two terms in the right-hand side of (A36) do not depend on p_T, so that they can be taken outside the integral in (A35). Hence, the integral to solve in (A35) is

$$\text{(A37)} \quad \int_{p_T} [p_T - r_{T-1}\,(p_{T-1} + 2\,\Theta\, I_T)]\, \frac{1}{\sigma_T \sqrt{2\,\pi}}$$

$$\exp\left\{-\frac{1}{2}\left[\frac{p_T - (\mu_T - \lambda\, \sigma_T^2\, I_T)}{\sigma_T}\right]^2\right\} dp_T = \mu_T - \lambda\, \sigma_T^2\, I_T - r_{T-1}\,(p_{T-1} + 2\,\Theta\, I_T)$$

Substituting (A35) through (A37) into (A34), and noting that $[-\lambda \exp(-\lambda\, r_{T-1}\, \pi_{T-1})] < 0$ and $\{-\exp[(\lambda\, \sigma_T\, I_T)^2/2] \exp(-\lambda\, \mu_T\, I_T)\} < 0$, yields

$$\text{(A38)}\quad \mu_T - r_{T-1}\, p_{T-1} - (2\, r_{T-1}\, \Theta + \lambda\, \sigma_T^2)\, I_T \le 0,$$

$$I_T \ge 0,\; I_T\, [\mu_T - r_{T-1}\, p_{T-1} - (2\, r_{T-1}\, \Theta + \lambda\, \sigma_T^2)\, I_T] = 0$$

Solving (A38) for I_T gives expression (2.22).

Example of Optimum Forward-Looking CARA Storage at Date T-2

According to (A33), the Kuhn-Tucker condition corresponding to I_{T-1} is

$$\text{(A39)}\quad \frac{\partial £_{T-2}}{\partial I_{T-1}} = -\lambda\, r_{T-1} \exp(-\lambda\, r_{T-2}\, r_{T-1}\, \pi_{T-2})$$

$$\mathbb{E}_{T-2}\{[p_{T-1} - r_{T-2}\, (p_{T-2} + i')] \max_{I_T \ge 0} £_{T-1}\} \le 0,\; I_{T-1} \ge 0,\; I_{T-1}\, \frac{\partial £_{T-2}}{\partial I_{T-1}} = 0$$

where: $£_{T-2} = \exp(-\lambda\, r_{T-2}\, r_{T-1}\, \pi_{T-2})\, \mathbb{E}_{T-2}(\max_{I_T \ge 0} £_{T-1})$

$£_{T-1} = \exp(-\lambda\, r_{T-1}\, \pi_{T-1})\, \mathbb{E}_{T-1}[-\exp(-\lambda\, p_T\, I_T)]$.

Expression (A39) can be rewritten as

$$\text{(A40)}\quad \mathbb{E}_{T-2}\{[p_{T-1} - r_{T-2}\, (p_{T-2} + 2\, \Theta\, I_{T-1})] \max_{I_T \ge 0} £_{T-1}\} \ge 0,\; I_{T-1} \ge 0,$$

$$I_{T-1}\, \mathbb{E}_{T-2}\{[p_{T-1} - r_{T-2}\, (p_{T-2} + 2\, \Theta\, I_{T-1})] \max_{I_T \ge 0} £_{T-1}\} = 0$$

because $[-\lambda\, r_{T-1} \exp(-\lambda\, r_{T-2}\, r_{T-1}\, \pi_{T-2})] < 0$ and $i'(I_{T-1}) = 2\,\Theta\, I_{T-1}$. But

$$\text{(A41)}\quad \mathbb{E}_{T-1}[-\exp(-\lambda\, p_T\, I_T)] = \int_{p_T} -\exp(-\lambda\, p_T\, I_T)\, dn(p_T)$$

$$= -\exp[(\lambda\, \sigma_T\, I_T)^2/2]\, \exp(-\lambda\, \mu_T\, I_T)$$

$$\int_{p_T} \frac{1}{\sigma_T \sqrt{2\,\pi}} \exp\left\{-\frac{1}{2}\left[\frac{p_T - (\mu_T - \lambda\, \sigma_T^2\, I_T)}{\sigma_T}\right]^2\right\} dp_T$$

$$= -\exp[(\lambda\, \sigma_T\, I_T)^2/2]\, \exp(-\lambda\, \mu_T\, I_T)$$

by application of expression (A36). Therefore,

$$\text{(A42)}\quad \max_{I_T \geq 0} \pounds_{T-1} = \max_{I_T \geq 0}\{\exp(-\lambda\, r_{T-1}\, \pi_{T-1})\, \mathbb{E}_{T-1}[-\exp(-\lambda\, p_T\, I_T)]\}$$

$$= \max_{I_T \geq 0}\{-\exp[-\lambda\, r_{T-1}\,(p_{T-1}\, I_{T-1} - p_{T-1}\, I_T - \Theta\, I_T^2)]$$

$$\exp[(\lambda\, \sigma_T\, I_T)^2/2]\, \exp(-\lambda\, \mu_T\, I_T)\}$$

$$= -\exp(-\lambda\, r_{T-1}\, p_{T-1}\, I_{T-1})$$

$$\exp[-\lambda\, I_T\,(\mu_T - r_{T-1}\, p_{T-1})]\, \exp[\lambda\, I_T^2\,(r_{T-1}\,\Theta + \lambda\, \sigma_T^2/2)]$$

where I_T is given by (2.22). Substitution of (A42) into the first inequality of (A40) yields

(A43) $\mathbb{E}_{T-2}\{[p_{T-1} - r_{T-2}(p_{T-2} + 2\,\Theta\, I_{T-1})]\ \max_{I_T \geq 0} \pounds_{T-1}\} =$

$$\int_{p_{T-1}} [p_{T-1} - r_{T-2}(p_{T-2} + 2\,\Theta\, I_{T-1})]\ [-\exp(-\lambda\, r_{T-1}\, p_{T-1}\, I_{T-1})]$$

$$\exp[-\lambda\, I_T(\mu_T - r_{T-1}\, p_{T-1})]\ \exp[\lambda\, I_T^2(r_{T-1}\,\Theta + \lambda\,\sigma_T^2/2)]\ dn(p_{T-1})$$

where $dn(p_{T-1}) = \dfrac{1}{\sigma_{T-1}\sqrt{2\,\pi}}\ \exp\left[-\dfrac{1}{2}\left(\dfrac{p_{T-1} - \mu_{T-1}}{\sigma_{T-1}}\right)^2\right] dp_{T-1}$

But

(A44) $-\exp(-\lambda\, r_{T-1}\, p_{T-1}\, I_{T-1})\ dn(p_{T-1}) = -\exp[-\lambda\, r_{T-1}\, I_{T-1}(\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1}/2)]$

$$\frac{1}{\sigma_{T-1}\sqrt{2\,\pi}}\ \exp\left\{-\frac{1}{2}\left[\frac{p_{T-1} - (\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1})}{\sigma_{T-1}}\right]^2\right\} dp_{T-1}$$

The first exponential term in the right-hand side of (A44) is negative everywhere and does not depend on p_{T-1}. Hence, the optimum I_{T-1} may be solved from a simplified version of (A40), namely,

(A45) $K_0 \leq 0,\ I_{T-1} \geq 0,\ I_{T-1}\, K_0 = 0$

where $K_0 = \displaystyle\int_{p_{T-1}} [p_{T-1} - r_{T-2}(p_{T-2} + 2\,\Theta\, I_{T-1})]\ \frac{1}{\sigma_{T-1}\sqrt{2\,\pi}}$

$$\exp[-\lambda\, I_T(\mu_T - r_{T-1}\, p_{T-1})]\ \exp[\lambda\, I_T^2(r_{T-1}\,\Theta + \lambda\,\sigma_T^2/2)]$$

$$\exp\left\{-\frac{1}{2}\left[\frac{p_{T-1} - (\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1})}{\sigma_{T-1}}\right]^2\right\} dp_{T-1}$$

Recalling that the optimum I_T is given by (2.22), we can rewrite K_0 as[46]

$$\text{(A46)}\quad K_0 = \int_0^{\mu_T/r_{T-1}} K_1 \exp[-\lambda\, I_T\,(\mu_T - r_{T-1}\, p_{T-1})] \exp[\lambda\, I_T^2\,(r_{T-1}\,\Theta + \lambda\,\sigma_T^2/2)]\, dp_{T-1} + \int_{\mu_T/r_{T-1}}^{\infty} K_1\, dp_{T-1}$$

$$\text{where } K_1 = \frac{[p_{T-1} - r_{T-2}\,(p_{T-2} + 2\,\Theta\, I_{T-1})]}{\sigma_{T-1}\sqrt{2\,\pi}} \exp\left\{-\frac{1}{2}\left[\frac{p_{T-1} - (\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1})}{\sigma_{T-1}}\right]^2\right\}$$

The second integral in the right-hand side of (A46) may be divided into two terms as follows

$$\text{(A47)}\quad \int_{\mu_T/r_{T-1}}^{\infty} K_1\, dp_{T-1} = \int_{\mu_T/r_{T-1}}^{\infty} \frac{[p_{T-1} - (\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1})]}{\sigma_{T-1}\sqrt{2\,\pi}} \exp\left\{-\frac{1}{2}\left[\frac{p_{T-1} - (\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1})}{\sigma_{T-1}}\right]^2\right\} dp_{T-1}$$

$$+ [\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1} - r_{T-2}\,(p_{T-2} + 2\,\Theta\, I_{T-1})]$$

$$\int_{\mu_T/r_{T-1}}^{\infty} \frac{1}{\sigma_{T-1}\sqrt{2\,\pi}} \exp\left\{-\frac{1}{2}\left[\frac{p_{T-1} - (\mu_{T-1} - \lambda\, r_{T-1}\,\sigma_{T-1}^2\, I_{T-1})}{\sigma_{T-1}}\right]^2\right\} dp_{T-1}$$

By application of the rules for truncated means of the normal distribution (Maddala, 1983) to the first term in (A47), and by noting that the integral in the second term is just the

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[46]Note that by the normality assumption the actual integration is from $-\infty$ to $+\infty$. However, in the numerical examples we will use parameters such that the probability of having $p_{T-1} < 0$ is negligible.

area under the normal distribution to the right of (μ_T/r_{T-1}), the solution to (A47) can be found as

$$\text{(A48)} \quad \int_{\mu_T/r_{T-1}}^{\infty} K_1 \, dp_{T-1} = \frac{1}{\sqrt{2\pi}} \exp[-\tfrac{1}{2}(\mu_T/r_{T-1})^2]$$

$$+ [\mu_{T-1} - \lambda r_{T-1} \sigma_{T-1}^2 I_{T-1} - r_{T-2}(p_{T-2} + 2\Theta I_{T-1})][1 - F(z_0)]$$

$$= \frac{1}{\sqrt{2\pi}} \exp[-\tfrac{1}{2}(\mu_T/r_{T-1})^2]$$

$$+ [\mu_{T-1} - r_{T-2} p_{T-2} - (2 r_{T-2}\Theta + \lambda r_{T-1}\sigma_{T-1}^2) I_{T-1}] F(-z_0)$$

where: $z_0 = -(\mu_{T-1} - \mu_T/r_{T-1} - \lambda r_{T-1} \sigma_{T-1}^2 I_{T-1})/\sigma_{T-1}$

$F(\cdot)$ = cumulative distribution function of the standard normal distribution

Applying the same strategy to the first right-hand side integral in (A46) yields

$$\text{(A49)} \quad \int_0^{\mu_T/r_{T-1}} K_1 \exp[-\lambda I_T(\mu_T - r_{T-1}p_{T-1})] \exp[\lambda I_T^2 (r_{T-1}\Theta + \lambda \sigma_T^2/2)] \, dp_{T-1}$$

$$= (1 + \sigma_{T-1}^2 K_2)^{-1/2} \exp(-\tfrac{1}{2} \sigma_{T-1}^2 K_2 z_1^2)$$

$$\{ \int_0^{\mu_T/r_{T-1}} \frac{[p_{T-1} - r_{T-2}(p_{T-2} + 2\Theta I_{T-1})]}{\sigma_{T-1}(1 + \sigma_{T-1}^2 K_2)^{-1/2}\sqrt{2\pi}}$$

$$\exp\left\{-\frac{1}{2}\left[\frac{p_{T-1}-(\mu_{T-1}-\lambda\, r_{T-1}\,\sigma_{T-1}^{\;2}\, I_{T-1}+\sigma_{T-1}^{\;2}\, K_2\,\mu_T/r_{T-1})/(1+\sigma_{T-1}^{\;2}\, K_2)}{\sigma_{T-1}\,(1+\sigma_{T-1}^{\;2}\, K_2)^{-1/2}}\right]^2\right\}$$

$$dp_{T-1}\}$$

$$= (1+\sigma_{T-1}^{\;2}\, K_2)^{-1/2}\exp(-\tfrac{1}{2}\,\sigma_{T-1}^{\;2}\, K_2\, z_1^{\;2})$$

$$\{\int_0^{\mu_T/r_{T-1}} \frac{[p_{T-1}-(\mu_{T-1}-\lambda\, r_{T-1}\,\sigma_{T-1}^{\;2}\, I_{T-1}+\sigma_{T-1}^{\;2}\, K_2\,\mu_T/r_{T-1})/(1+\sigma_{T-1}^{\;2}\, K_2)]}{\sigma_{T-1}\,(1+\sigma_{T-1}^{\;2}\, K_2)^{-1/2}\sqrt{2\,\pi}}$$

$$\exp\left\{-\frac{1}{2}\left[\frac{p_{T-1}-(\mu_{T-1}-\lambda\, r_{T-1}\,\sigma_{T-1}^{\;2}\, I_{T-1}+\sigma_{T-1}^{\;2}\, K_2\,\mu_T/r_{T-1})/(1+\sigma_{T-1}^{\;2}\, K_2)}{\sigma_{T-1}\,(1+\sigma_{T-1}^{\;2}\, K_2)^{-1/2}}\right]^2\right\}$$

$$dp_{T-1}$$

$$+\,[(\mu_{T-1}-\lambda\, r_{T-1}\,\sigma_{T-1}^{\;2}\, I_{T-1}+\sigma_{T-1}^{\;2}\, K_2\,\mu_T/r_{T-1})/(1+\sigma_{T-1}^{\;2}\, K_2)-r_{T-2}\,(p_{T-2}+2\,\Theta\, I_{T-1})]$$

$$\int_0^{\mu_T/r_{T-1}} \frac{1}{\sigma_{T-1}\,(1+\sigma_{T-1}^{\;2}\, K_2)^{-1/2}\sqrt{2\,\pi}}$$

$$\exp\left\{-\frac{1}{2}\left[\frac{p_{T-1}-(\mu_{T-1}-\lambda\, r_{T-1}\,\sigma_{T-1}^{\;2}\, I_{T-1}+\sigma_{T-1}^{\;2}\, K_2\,\mu_T/r_{T-1})/(1+\sigma_{T-1}^{\;2}\, K_2)}{\sigma_{T-1}\,(1+\sigma_{T-1}^{\;2}\, K_2)^{-1/2}}\right]^2\right\}$$

$$dp_{T-1}\}$$

$$= z_1/z_0 \exp(-\tfrac{1}{2}\ \sigma_{T-1}{}^2 K_2\, z_1{}^2)\ \{-\ \frac{1}{\sqrt{2\ \pi}}\ \ \exp[-\tfrac{1}{2}(\mu_T/r_{T-1})^2]$$

$$+ [(\mu_{T-1} - \lambda\, r_{T-1}\, \sigma_{T-1}{}^2\, I_{T-1} + \sigma_{T-1}{}^2 K_2\, \mu_T/r_{T-1})\ (z_1/z_0)^2 - r_{T-2}\ (p_{T-2} + 2\ \Theta\ I_{T-1})]\ F(z_1)\}$$

where: $K_2 = r_{T-1}{}^2\ \lambda/(2\ r_{T-1}\ \Theta + \lambda\ \sigma_T{}^2)$

$z_1 = (1 + \sigma_{T-1}{}^2\ K_2)^{-1/2}\ z_0$

Substituting (A48) and (A49) back into (A46) and rearranging gives (2.23), which is the final expression to solve for I_{T-1}.

Proof of Propositions 2.5 and 2.6

We will only outline the proof of Propositions 2.5 and 2.6 because they can be done employing the same techniques we used to show Propositions 2.3 and 2.4, respectively.

We may express FOC (2.28) in covariance terms as

$$\text{(A50)}\quad \mathbb{E}_0(p_1) + \frac{\mathrm{Cov}[p_1;\ \mathbb{E}_0(M_1{}'|\ p_1)]}{M_0{}'}\ \begin{cases} = r_0\ [\Phi\ s_0\ +\ c'(Q_0)]\ \text{if}\ Q_0 > 0 \\ \le r_0\ [\Phi\ s_0\ +\ c'(0)]\ \text{if}\ Q_0\ =\ 0 \end{cases}$$

where: $\mathbb{E}_0(M_1{}'|\ p_1) = \int_{s_1} M_1{}'(W_T,\ e_1)\ s_1(s_1|\ p_0,\ s_0,\ p_1)\ ds_1 > 0$

$\mathbb{E}_0(M_1{}'|\ p_1)$ is the conditional expectation of $M_1{}'$ given p_1, and $s_1(s_1|\ p_0,\ s_0,\ p_1)$ denotes the conditional density function of s_1 given $(p_0,\ s_0,\ p_1)$. Expression (A50) is analogous to (2.11) and (2.12) for the risk-averse productive firm. Similarly, a risk-neutral productive firm is characterized by:

$$\text{(A51)}\quad \mathbb{E}_0(p_1) \begin{cases} = r_0\,[\Phi\, s_0 + c'(Q_0)] \text{ if } Q_0 > 0 \\ \leq r_0\,[\Phi\, s_0 + c'(0)] \text{ if } Q_0 = 0 \end{cases}$$

For a myopic risk-averse decision maker we have $\mathbb{E}_0(M_1' | p_1) = M_T'$, and

$$\text{(A52)}\quad \frac{\partial \mathbb{E}_0(M_1' | p_1)}{\partial p_1} = Q_{T-1}\, M_T'' \begin{cases} < 0 \text{ if } Q_{T-1} > 0 \\ = 0 \text{ if } Q_{T-1} = 0 \end{cases}$$

Therefore,

$$\text{(A53)}\quad \mathrm{Cov}(p_T, M_T') \begin{cases} < 0 \text{ if } Q_{T-1} > 0 \\ = 0 \text{ if } Q_{T-1} = 0 \end{cases}$$

Proposition 2.5 follows immediately by noting that (A53) is analogous to (2.14).

For a nonmyopic CARA firm with output and material input prices related as in (2.29) or (2.30), we have

$$\text{(A54)}\quad \frac{\partial \mathbb{E}_0(M_1' | p_1)}{\partial p_1} = r_1 \ldots r_{T-1}\, Q_0\, \mathbb{E}_0(M_1'' | p_1) - r_1 \ldots r_{T-1}\, \Phi\, \mathbb{E}_0(Q_1\, M_1'' \frac{\partial s_1}{\partial p_1} \,|\, p_1)$$

$$+ \max_{Q_1 \geq 0}[\int_{p_2}\int_{s_2} M_2' \frac{\partial g_2(p_2, s_2 | p_0, s_0, p_1, s_1)}{\partial p_1}\, ds_2\, dp_2]$$

$$+ \ldots + \max_{Q_1 \geq 0}\{\int_{p_2}\int_{s_2} \max_{Q_2 \geq 0}[\int_{p_3}\int_{s_3} \ldots$$

$$\int_{p_{T-1}}\int_{s_{T-1}} \max_{Q_{T-1} \geq 0}(\int_{p_T}\int_{s_T} M_T' \frac{\partial g_T(p_T, s_T | p_0, s_0, \ldots, p_{T-1}, s_{T-1})}{\partial p_1}\, ds_T\, dp_T)$$

$$g_{T-1}(p_{T-1}, s_{T-1} | p_0, s_0, \ldots, p_{T-2}, s_{T-2})\, ds_{T-1}\, dp_{T-1}] \ldots g_2(p_2, s_2 | p_0, s_0, p_1, s_1)\, ds_2\, dp_2\}$$

The term $[r_1 \ldots r_{T-1} Q_0 \mathbb{E}_0(M_1'' | p_1)]$ is negative if $Q_0 > 0$, and zero if $Q_0 = 0$. The term $[- r_1 \ldots r_{T-1} \Phi \mathbb{E}_0(Q_1 M_1'' \partial s_1/\partial p_1 | p_1)]$ is negative because $Q_1 \geq 0$, $M_1'' < 0$, and $\partial s_1/\partial p_1 > 0$. Finally, the terms $\max_{Q_1 \geq 0}(\cdot)$ have ambiguous signs.

Expression (A54) is analogous to (2.13) for the CARA case, so that the proofs under assumptions (a), (b), and (c) in Proposition 2.6 are straightforward.

APPENDIX B. APPENDIX TO CHAPTER III

Proof that $\mathbb{E}_t(I_{t+2}\, M_{t+1}''|\, p_{t+1}) < 0$

From the proof of Proposition 3.1 we know that $I_{t+2} = 0$ if $f_{t+1} \leq r_{t+1}\,[p_{t+1} + i'(0)]$, and $I_{t+2} > 0$ otherwise. Hence,

$$\text{(B1)} \quad \mathbb{E}_t(I_{t+2}\, M_{t+1}''|\, p_{t+1}) = \int_{f_{t+1}} I_{t+2}\, M_{t+1}''\, f_{t+1}(f_{t+1}|\, p_0, f_0, \ldots, p_{t+1})\, df_{t+1}$$

$$= \int_{r_{t+1}[p_{t+1}+i'(0)]}^{\infty} I_{t+2}\, M_{t+1}''\, f_{t+1}(f_{t+1}|\, p_0, f_0, \ldots, p_{t+1})\, df_{t+1}$$

But $M_{t+1}'' < 0$, so that $\mathbb{E}_t(I_{t+2}\, M_{t+1}''|\, p_{t+1}) < 0$ if the probability that $f_{t+1} > r_{t+1}\,[p_{t+1} + i'(0)]$ is positive, i.e., if

$$\text{(B2)} \quad \int_{r_{t+1}[p_{t+1}+i'(0)]}^{\infty} f_{t+1}(f_{t+1}|\, p_0, f_0, \ldots, p_{t+1})\, df_{t+1} > 0$$

Proof of Proposition 3.4

By substitution of FOC (3.21) into FOC (3.20) and rearrangement we obtain

$$\text{(B3)} \quad f_t - r_t\,[\Phi\, s_t + c'(Q_t)] \leq 0,\; Q_t \geq 0,\; Q_t\,\{f_t - r_t\,[\Phi\, s_t + c'(Q_t)]\} = 0$$

Therefore,

a. If $f_t \leq r_t\,[\Phi\, s_t + c'(0)]$, then $Q_t = 0$.

b. If $f_t > r_t\,[\Phi\, s_t + c'(0)]$, then $Q_t > 0$ and $f_t = r_t\,[\Phi\, s_t + c'(Q_1)]$.

Proof of Proposition 3.5

Rewrite FOCs (3.21) and (3.22) as

(B4) $[f_0 - E_0(p_1)]\, M_0' = \mathrm{Cov}[p_1, E_0(M_1'|\, p_1)]$

(B5) $[f_0^s - E_0(s_1)]\, M_0' = \mathrm{Cov}[s_1, E_0(M_1'|\, s_1)]$

where: $E_0(M_1'|\, p_1) = \int_{s_1}\int_{f_1}\int_{f_1^s} M_1'\, k_1(s_1, f_1, f_1^s|\, p_0, s_0, f_0, f_0^s, p_1)\, df_1^s\, df_1\, ds_1 > 0$

$E_0(M_1'|\, s_1) = \int_{p_1}\int_{f_1}\int_{f_1^s} M_1'\, l_1(p_1, f_1, f_1^s|\, p_0, s_0, f_0, f_0^s, s_1)\, df_1^s\, df_1\, dp_1 > 0$

The function $k_1(s_1, f_1, f_1^s|\, p_0, s_0, f_0, f_0^s, p_1)$ is the conditional density of s_1, f_1, and f_1^s given $(p_0, s_0, f_0, f_0^s, p_1)$, and $l_1(p_1, f_1, f_1^s|\, p_0, s_0, f_0, f_0^s, s_1)$ is the conditional density function of p_1, f_1, and f_1^s given $(p_0, s_0, f_0, f_0^s, s_1)$.

If the firm is myopic and both forward prices are simultaneously unbiased, we need $\mathrm{Cov}[p_T, E_{T-1}(M_T'|\, p_T)] = \mathrm{Cov}[s_T, E_{T-1}(M_T'|\, s_T)] = 0$. This is satisfied if $F_{T-1} = Q_{T-1}$ and $F_{T-1}^s = 0$ because such hedge yields $M_1' = M_T'$ independent from both p_T and s_T.

Proof of Proposition 3.6

In the case of a forward-looking CARA firm we have

(B6) $$\frac{\partial E_0(M_1'|\, p_1)}{\partial p_1} = r_1 \ldots r_{T-1}\,(Q_0 - F_0)\, E_0(M_1''|\, p_1)$$

$$- \int_{s_1} \int_{f_1} \int_{f_1^s} \frac{M_1''}{\lambda} \frac{\partial k_1(s_1, f_1, f_1^s \mid p_0, s_0, f_0, f_0^s, p_1)}{\partial p_1} \, df_1^s \, df_1 \, ds_1$$

$$- \int_{s_1} \int_{f_1} \int_{f_1^s} \{\max_{\mathbf{d}_1}[\int_{p_2} \int_{s_2} \int_{f_2} \int_{f_2^s} \frac{M_2''}{\lambda}$$

$$\frac{\partial j_2(p_2, s_2, f_2, f_2^s \mid p_0, s_0, f_0, f_0^s, p_1, s_1, f_1, f_1^s)}{\partial p_1} \, df_2^s \, df_2 \, ds_2 \, dp_2]\}$$

$$k_1(s_1, f_1, f_1^s \mid p_0, s_0, f_0, f_0^s, p_1) \, df_1^s \, df_1 \, ds_1$$

$$- \ldots - \int_{s_1} \int_{f_1} \int_{f_1^s} \{\max_{\mathbf{d}_1}[\int_{p_2} \int_{s_2} \int_{f_2} \int_{f_2^s} \max_{\mathbf{d}_2}(\int_{p_3} \int_{s_3} \int_{f_3} \int_{f_3^s} \ldots$$

$$\int_{p_{T-1}} \int_{s_{T-1}} \int_{f_{T-1}} \int_{f_{T-1}^s} \max_{\mathbf{d}_{T-1}}(\int_{p_T} \int_{s_T} \int_{f_T} \int_{f_T^s} \frac{M_T''}{\lambda}$$

$$\frac{\partial j_T(p_T, s_T, f_T, f_T^s \mid p_0, s_0, f_0, f_0^s, \ldots, p_{T-1}, s_{T-1}, f_{T-1}, f_{T-1}^s)}{\partial p_1} \, df_T^s \, df_T \, ds_T \, dp_T)$$

$$j_{T-1}(p_{T-1}, s_{T-1}, f_{T-1}, f_{T-1}^s \mid p_0, s_0, f_0, f_0^s, \ldots, p_{T-2}, s_{T-2}, f_{T-2}, f_{T-2}^s)$$

$$df_{T-1}^s \, df_{T-1} \, ds_{T-1} \, dp_{T-1}) \ldots$$

$$j_2(p_2, s_2, f_2, f_2^s \mid p_0, s_0, f_0, f_0^s, p_1, s_1, f_1, f_1^s) \, df_2^s \, df_2 \, ds_2 \, dp_2]\}$$

$$k_1(s_1, f_1, f_1^s \mid p_0, s_0, f_0, f_0^s, p_1)\, df_1^s\, df_1\, ds_1$$

(B7) $$\frac{\partial \mathbb{E}_0(M_1' \mid s_1)}{\partial s_1} = - r_1 \dots r_{T-1} F_0^s\, \mathbb{E}_0(M_1'' \mid s_1) - r_1 \dots r_{T-1}\, \Phi\, \mathbb{E}_0(Q_1 M_1'' \mid s_1)$$

$$- \int_{p_1} \int_{f_1} \int_{f_1^s} \frac{M_1''}{\lambda} \frac{\partial l_1(p_1, f_1, f_1^s \mid p_0, s_0, f_0, f_0^s, s_1)}{\partial s_1}\, df_1^s\, df_1\, dp_1$$

$$- \int_{p_1} \int_{f_1} \int_{f_1^s} \{\max_{d_1} [\int_{p_2} \int_{s_2} \int_{f_2} \int_{f_2^s} \frac{M_2''}{\lambda}$$

$$\frac{\partial j_2(p_2, s_2, f_2, f_2^s \mid p_0, s_0, f_0, f_0^s, p_1, s_1, f_1, f_1^s)}{\partial s_1}\, df_2^s\, df_2\, ds_2\, dp_2]\}$$

$$l_1(p_1, f_1, f_1^s \mid p_0, s_0, f_0, f_0^s, s_1)\, df_1^s\, df_1\, dp_1$$

$$- \dots - \int_{p_1} \int_{f_1} \int_{f_1^s} \{\max_{d_1} [\int_{p_2} \int_{s_2} \int_{f_2} \int_{f_2^s} \max_{d_2} (\int_{p_3} \int_{s_3} \int_{f_3} \int_{f_3^s} \dots$$

$$\int_{p_{T-1}} \int_{s_{T-1}} \int_{f_{T-1}} \int_{f_{T-1}^s} \max_{d_{T-1}} (\int_{p_T} \int_{s_T} \int_{f_T} \int_{f_T^s} \frac{M_T''}{\lambda}$$

$$\frac{\partial j_T(p_T, s_T, f_T, f_T^s \mid p_0, s_0, f_0, f_0^s, \dots, p_{T-1}, s_{T-1}, f_{T-1}, f_{T-1}^s)}{\partial s_1}\, df_T^s\, df_T\, ds_T\, dp_T)$$

$$j_{T-1}(p_{T-1}, s_{T-1}, f_{T-1}, f_{T-1}^s \mid p_0, s_0, f_0, f_0^s, \dots, p_{T-2}, s_{T-2}, f_{T-2}, f_{T-2}^s)$$

$$df^S_{T-1}\, df_{T-1}\, ds_{T-1}\, dp_{T-1}) \ldots$$

$$j_2(p_2, s_2, f_2, f^S_2 | p_0, s_0, f_0, f^S_0, p_1, s_1, f_1, f^S_1)\, df^S_2\, df_2\, ds_2\, dp_2]\}$$

$$l_1(p_1, f_1, f^S_1 | p_0, s_0, f_0, f^S_0, s_1)\, df^S_1\, df_1\, dp_1$$

Under assumption (a), expressions (B6) and (B7) simplify respectively to

$$\text{(B8)} \quad \frac{\partial \mathbb{E}_0(M_1' | p_1)}{\partial p_1} = r_1 \ldots r_{T-1} (Q_0 - F_0)\, \mathbb{E}_0(M_1'' | p_1)$$

$$\text{(B9)} \quad \frac{\partial \mathbb{E}_0(M_1' | s_1)}{\partial s_1} = - r_1 \ldots r_{T-1} F^S_0\, \mathbb{E}_0(M_1'' | s_1) - r_1 \ldots r_{T-1}\, \Phi\, \mathbb{E}_0(Q_1 M_1'' | s_1)$$

The FOCs require $\mathrm{Cov}[p_1, \mathbb{E}_0(M_1' | p_1)] = \mathrm{Cov}[s_1, \mathbb{E}_0(M_1' | s_1)] = 0$ under unbiased forward prices. This condition is met if $\partial \mathbb{E}_0(M_1' | p_1)/\partial p_1 = \partial \mathbb{E}_0(M_1' | s_1)/\partial s_1 = 0$, which requires setting $F_0 = Q_0$ and $F^S_0 < 0$.

The result under assumption (b) follows because

$$\text{(B10)} \quad \lim_{\lambda \to \infty} \frac{\partial \mathbb{E}_0(M_1' | p_1)}{\partial p_1} = r_1 \ldots r_{T-1} (Q_0 - F_0)\, \mathbb{E}_0(M_1'' | p_1)$$

$$\text{(B11)} \quad \lim_{\lambda \to \infty} \frac{\partial \mathbb{E}_0(M_1' | s_1)}{\partial s_1} = - r_1 \ldots r_{T-1} F^S_0\, \mathbb{E}_0(M_1'' | s_1)$$

$$- r_1 \ldots r_{T-1}\, \Phi\, \mathbb{E}_0(Q_1 M_1'' | s_1)$$

Proof of Proposition 3.7

In the absence of a forward market for material input, the dynamic programming problem to solve for the optimal decision vector is

(B12) $M_t(W_T, e_t) = \max_{d_t} £_t(W_T) | e_t$

where: $£_T(W_T) = U(W_T)$

$$£_t(W_T) = \int_{p_{t+1}} \int_{s_{t+1}} \int_{f_{t+1}} M_{t+1}(W_T, e_{t+1})$$

$$m_{t+1}(p_{t+1}, s_{t+1}, f_{t+1} | p_0, s_0, f_0, \ldots, p_t, s_t, f_t)\, df_{t+1}\, ds_{t+1}\, dp_{t+1},\ 0 \le t < T$$

$$\mathbf{d}_t = (Q_t, F_t) \text{ if } 0 \le t < T,\ \mathbf{d}_T = (Q_T, 0),\ Q_t \ge 0$$

$$\pi_t = p_t\, Q_{t-1} - \Phi\, s_t\, Q_t - c(Q_t) + (f_{t-1} - p_t)\, F_{t-1}$$

Terminal wealth is given by (2.1), and $m_{t+1}(p_{t+1}, s_{t+1}, f_{t+1} | p_0, s_0, f_0, \ldots, p_t, s_t, f_t)$ is the conditional density function of p_{t+1}, s_{t+1}, and f_{t+1} given $(p_0, s_0, f_0, \ldots, p_t, s_t, f_t)$. The maximum attainable utility at the terminal date is

(B13) $M_T(W_T, e_T) = U[r_{T-1}\, W_{T-1} + p_T\, Q_{T-1} + (f_{T-1} - p_T)\, F_{T-1}]$

and the FOCs for dates $0 \le t < T$ are

(B14) $\dfrac{\partial £_t}{\partial Q_t} = r_{t+1} \cdots r_{T-1}\, [\mathbb{E}_t(p_{t+1}\, M_{t+1}') - r_t\, (\Phi\, s_t + c')\, M_t'] \le 0,\ Q_t \ge 0,\ Q_t\, \dfrac{\partial £_t}{\partial Q_t} = 0$

(B15) $$\frac{\partial \pounds_t}{\partial F_t} = r_{t+1} \cdots r_{T-1} [f_t M_t' - \mathbb{E}_t(p_{t+1} M_{t+1}')] = 0$$

Expression (B15) can be rewritten as

(B16) $$[f_0 - \mathbb{E}_0(p_1)] M_0' = \text{Cov}[p_1, \mathbb{E}_0(M_1' | p_1)]$$

where: $\mathbb{E}_0(M_1' | p_1) = \int_{s_1} \int_{f_1} M_1' \, n_1(s_1, f_1 | p_0, s_0, f_0, p_1) \, df_1 \, ds_1 > 0$

The function $n_1(s_1, f_1 | p_0, s_0, f_0, p_1)$ is the conditional density of s_1 and f_1 given (p_0, s_0, f_0, p_1). If the firm is CARA and cash prices are related as in (2.29) or (2.30), we have

(B17) $$\frac{\partial \mathbb{E}_0(M_1' | p_1)}{\partial p_1} = r_1 \ldots r_{T-1} (Q_0 - F_0) \, \mathbb{E}_0(M_1'' | p_1)$$

$$- r_1 \ldots r_{T-1} \, \Phi \, \mathbb{E}_0(Q_1 M_1'' \frac{\partial s_1}{\partial p_1} | p_1)$$

$$- \int_{f_1} \frac{M_1''}{\lambda} \frac{\partial o_1(f_1 | p_0, s_0, f_0, p_1)}{\partial p_1} \, df_1$$

$$- \int_{f_1} \{\max_{d_1}[\int_{p_2} \int_{s_2} \int_{f_2} \frac{M_2''}{\lambda} \frac{\partial m_2(p_2, s_2, f_2 | p_0, s_0, f_0, p_1, s_1, f_1)}{\partial p_1} \, df_2 \, ds_2 \, dp_2]\}$$

$$o_1(f_1 | p_0, s_0, f_0, p_1) \, df_1$$

$$- \ldots - \int_{f_1} \{\max_{d_1}[\int_{p_2} \int_{s_2} \int_{f_2} \max_{d_2}(\int_{p_3} \int_{s_3} \int_{f_3} \cdots \int_{p_{T-1}} \int_{s_{T-1}} \int_{f_{T-1}}$$

$$\max_{d_{T-1}} \left(\int_{p_T} \int_{s_T} \int_{f_T} \frac{M_T''}{\lambda} \right.$$

$$\frac{\partial m_T(p_T, s_T, f_T \mid p_0, s_0, f_0, \ldots, p_{T-1}, s_{T-1}, f_{T-1})}{\partial p_1} \, df_T \, ds_T \, dp_T)$$

$$m_{T-1}(p_{T-1}, s_{T-1}, f_{T-1} \mid p_0, s_0, f_0, \ldots, p_{T-2}, s_{T-2}, f_{T-2}) \, df_{T-1} \, ds_{T-1} \, dp_{T-1}) \ldots$$

$$m_2(p_2, s_2, f_2 \mid p_0, s_0, f_0, p_1, s_1, f_1) \, df_2 \, ds_2 \, dp_2]\} \, o_1(f_1 \mid p_0, s_0, f_0, p_1) \, df_1$$

where $o_1(f_1 \mid p_0, s_0, f_0, p_1)$ is the conditional density function of f_1 given (p_0, s_0, f_0, p_1).

Under assumption (a), expression (B17) reduces to

$$\text{(B18)} \quad \frac{\partial \mathbb{E}_0(M_1' \mid p_1)}{\partial p_1} = r_1 \ldots r_{T-1} (Q_0 - F_0) \, \mathbb{E}_0(M_1'' \mid p_1)$$

$$- r_1 \ldots r_{T-1} \, \Phi \, \mathbb{E}_0(Q_1 \, M_1'' \frac{\partial s_1}{\partial p_1} \mid p_1)$$

which is positive for $F_0 \geq Q_0$. Unbiased forward price requires $\mathrm{Cov}[p_1, \mathbb{E}_0(M_1' \mid p_1)] = 0$, which means that it is necessary to have $F_0 < Q_0$ (otherwise, $\mathrm{Cov}[p_1, \mathbb{E}_0(M_1' \mid p_1)] > 0$).

The result under assumption (b) follows from the fact that

$$\text{(B19)} \quad \lim_{\lambda \to \infty} \frac{\partial \mathbb{E}_0(M_1' \mid p_1)}{\partial p_1} = r_1 \ldots r_{T-1} (Q_0 - F_0) \, \mathbb{E}_0(M_1'' \mid p_1)$$

$$- r_1 \ldots r_{T-1} \, \Phi \, \mathbb{E}_0(Q_1 \, M_1'' \frac{\partial s_1}{\partial p_1} \mid p_1)$$

APPENDIX C. APPENDIX TO CHAPTER IV

Derivation of FOCs (4.3) through (4.5)

The FOCs corresponding to the Lagrangian for t < T are

$$\text{(C1)}\quad \frac{\partial \pounds_t}{\partial P_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial P_t}\right]$$

$$+ r_t r_{t+1} \cdots r_{T-2} r_{T-1} (p_t + i') M_t'(W_T, e_t) - \eta_t = 0$$

$$\text{(C2)}\quad \frac{\partial \pounds_t}{\partial Q_t^s} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial Q_t} \frac{\partial Q_t}{\partial Q_t^s} + \frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}^s} \frac{\partial I_{t+1}^s}{\partial Q_t^s}\right]$$

$$- r_t r_{t+1} \cdots r_{T-2} r_{T-1} (c'/\Phi - i') M_t'(W_T, e_t) - \eta_t^s \le 0,\ Q_t^s \ge 0,\ Q_t^s \frac{\partial \pounds_t}{\partial Q_t^s} = 0$$

$$\text{(C3)}\quad \frac{\partial \pounds_t}{\partial S_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}^s} \frac{\partial I_{t+1}^s}{\partial S_t}\right]$$

$$- r_t r_{t+1} \cdots r_{T-2} r_{T-1} (s_t + i^{s\prime}) M_t'(W_T, e_t) + \eta_t^s = 0$$

plus (4.6) through (4.9). But first note that

$$\text{(C4)}\quad \frac{\partial I_{t+1}}{\partial P_t} = -1$$

$$\text{(C5)}\quad \frac{\partial I_{t+1}}{\partial I_t} = 1$$

$$\text{(C6)} \quad \frac{\partial I_{t+1}}{\partial Q_t} = 1$$

$$\text{(C7)} \quad \frac{\partial I^s_{t+1}}{\partial S_t} = 1$$

$$\text{(C8)} \quad \frac{\partial I^s_{t+1}}{\partial I^s_t} = 1$$

$$\text{(C9)} \quad \frac{\partial I^s_{t+1}}{\partial Q^s_t} = -1$$

$$\text{(C10)} \quad \frac{\partial Q_t}{\partial Q^s_t} = 1/\Phi$$

In addition,

$$\text{(C11)} \quad \frac{\partial M_t(W_T, e_t)}{\partial I_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} \; \frac{\partial I_{t+1}}{\partial I_t}\right] - r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, i'\, M_t'(W_T, e_t) + \eta_t$$

$$= r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, p_t\, M_t'(W_T, e_t)$$

$$\text{(C12)} \quad \frac{\partial M_t(W_T, e_t)}{\partial I^s_t} = \mathbb{E}_t\left[\frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I^s_{t+1}} \; \frac{\partial I^s_{t+1}}{\partial I^s_t}\right] - r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, i^{S}{}'\, M_t'(W_T, e_t) + \eta^s_t$$

$$= r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, s_t\, M_t'(W_T, e_t)$$

where the second equalities in (C11) and (C12) are derived by employing expressions (C1), (C3), (C4), (C5), (C7), and (C8). From (C11) and (C12), it follows that

$$\text{(C13)} \quad \frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I_{t+1}} = r_{t+1}\, r_{t+2} \cdots r_{T-2}\, r_{T-1}\, p_{t+1}\, M_{t+1}'(W_T, e_{t+1})$$

$$\text{(C14)} \quad \frac{\partial M_{t+1}(W_T, e_{t+1})}{\partial I^{s}_{t+1}} = r_{t+1}\, r_{t+2} \cdots r_{T-2}\, r_{T-1}\, s_{t+1}\, M_{t+1}'(W_T, e_{t+1})$$

Finally, by substituting (C4)-(C10) and (C13)-(C14) into FOCs (C1)-(C3), and by rearranging a little, we get expressions (4.3) through (4.5).

Rationale for Including Production as an Explanatory Variable of Output Sales

For simplicity, assume that at date t optimal production and sales levels are Q_t^* and zero, respectively. Let all exogenous variables stay unchanged for the remainder j decision dates comprised in the observation horizon O. It follows from expression (4.11) that optimal production for all decision dates t+1 through t+j will remain unchanged, so that production over the observation horizon will be

$$\text{(C15)} \quad Q_O = j\, Q_t^*$$

According to expression (4.10), optimal sales in the decision dates t+1 through t+j will be identical to the changes in beginning stocks, which are equal to optimal production in the previous decision date (i.e., $P_t^* = 0$, and $P_{t+1}^* = P_{t+2}^* = \ldots = P_{t+j}^* = Q_t^*$). Hence, sales over the observation horizon are

$$\text{(C16)} \quad P_O = (j - 1)\, Q_t^* = (1 - 1/j)\, Q_O$$

According to (C16), observed sales (P_O) approach asymptotically observed production (Q_O) as the observation horizon lengthens with respect to the decision horizon (so that $j \rightarrow \infty$). This means that in the regression corresponding to sales we must include production as an explanatory variable if $j > 1$, even though both quantities are endogenous. Note also that the effect is from production on sales and not the other way around, so that sales ought not be included as an explanatory variable of production.

We can apply the same reasoning to motivate the inclusion of processing as an explanatory variable in the regression for material input purchases.

Restrictions and Identities Corresponding to Equations (4.13) through (4.16)

The identities and restrictions for the soybean complex are, respectively,

$$\text{(C17)} \quad I_t^s = I_{t-1}^s + S_{t-1} - Q_{t-1}^s \qquad \text{Soybean Stocks}$$

$$\text{(C18)} \quad I_t^o = I_{t-1}^o - P_{t-1}^o + Q_{t-1}^o \qquad \text{Oil Stocks}$$

$$\text{(C19)} \quad I_t^m = I_{t-1}^m - P_{t-1}^m + Q_{t-1}^m \qquad \text{Meal Stocks}$$

$$\text{(C20)} \quad Q_t^o = Q_t^s/\Phi^o \qquad \text{Oil Production}$$

$$\text{(C21)} \quad Q_t^m = Q_t^s/\Phi^m \qquad \text{Meal Production}$$

Expressions for RETURN Variables

The specification of the RETURN variables is as follows:

$$\text{(C22)}\quad RETURN_t^c = 1/r_t\,[(f_{t;t+k}^o/\Phi^o + f_{t;t+k}^m/\Phi^m)/s_t]^{12/k}$$

$$\text{(C23)}\quad RETURN_t^s = 1/r_t\,(f_{t;t+h}^s/s_t)^{12/h}$$

$$\text{(C24)}\quad RETURN_t^o = 1/r_t\,(f_{t;t+k}^o/p_t^o)^{12/k}$$

$$\text{(C25)}\quad RETURN_t^m = 1/r_t\,(f_{t;t+k}^m/p_t^m)^{12/k}$$

k is 2 for t = January, March, May, June, October, and November; it is 3 for t = February, April, September, and December; it is 4 for t = August; and it is 5 for t = July. h is 2 for t = January, March, May, June, September, and November; it is 3 for t = February, April, August, and October; and it is 4 for t = July. As inferred from information on open interest, on average these are the most used combinations of hedge-placement/delivery months.

Derivation of Long-Term Elasticities from Structural Parameters

In a long-term equilibrium the beginning stocks must remain unchanged from date to date, hence

$$\text{(C26)}\quad I_t^s = I_{t-1}^s = I^s \qquad \Rightarrow \qquad S = Q^s$$

$$\text{(C27)}\quad I_t^o = I_{t-1}^o = I^o \qquad \Rightarrow \qquad P^o = Q^o = Q^s/\Phi^o$$

$$\text{(C28)} \quad I_t^m = I_{t-1}^m = I^m \qquad \Rightarrow \qquad P^m = Q^m = Q^s/\Phi^m$$

where we drop the time subscripts when we refer to long-term values.

From the regression for oil sales we have

$$\text{(C29)} \quad \ln(P^o) = \alpha_1 - 0.105 \ln(RETURN^o) + 0.115 \ln(I^o) + 0.697 \ln(Q^o)$$

where α_i ($i = 1, 2, \ldots, 6$) represents terms in the regression that are irrelevant for our purposes. Substituting (C27) into (C29) and solving for $\ln(I^o)$ in terms of $\ln(Q^s)$ yields

$$\text{(C30)} \quad \ln(I^o) = \alpha_2 + 0.913 \ln(RETURN^o) + 2.626 \ln(Q^s)$$

By performing analogous operations for meal sales and soybean purchases we get

$$\text{(C31)} \quad \ln(I^m) = \alpha_3 + 0.742 \ln(RETURN^m) + 1.008 \ln(Q^s)$$

$$\text{(C32)} \quad \ln(I^s) = \alpha_4 + 1.114 \ln(RETURN^c) + 0.533 \ln(RETURN^s) + 0.846 \ln(Q^s) + 0.061 \ln(I^o) - 0.184 \ln(I^m)$$

Finally, by replacing (C30) and (C31) into the regression for crushings (C33)

$$\text{(C33)} \quad \ln(Q^s) = \alpha_5 + 0.075 \ln(RETURN^c) - 0.0256 \ln(I^o) - 0.052 \ln(I^m) + 0.874 \ln(Q^s) - 0.102 \ln(Q^s) + 0.291 \ln(CAP)$$

and solving this for $\ln(Q^S)$ we obtain

$$\text{(C34)}\quad \ln(Q^S) = \alpha_6 + 0.216\ \ln(\text{RETURN}^c) - 0.067\ \ln(\text{RETURN}^o)$$

$$- 0.111\ \ln(\text{RETURN}^m) + 0.835\ \ln(\text{CAP})$$

Expression (C34) is the basic one to calculate the long-term equilibrium elasticities for crushings. Substitution of (C34) in (C30)-(C32) allows us to obtain the long-term elasticities for inventories. The mean values used in the calculations were $(f^o_{t;t+k}/\Phi^o)/(f^o_{t;t+k}/\Phi^o + f^m_{t;t+k}/\Phi^m) = 0.372$, and $(f^m_{t;t+k}/\Phi^m)/(f^o_{t;t+k}/\Phi^o + f^m_{t;t+k}/\Phi^m) = 0.628$.

Expressions for PERFOR and NAIVE Variables

The PERFOR and NAIVE variables are defined as follows:

$$\text{(C35)}\quad \text{PERFOR}^c_t = 1/r_t\ [(p^o_{t+k}/\Phi^o + p^m_{t+k}/\Phi^m)/s_t]^{12/k}$$

$$\text{(C36)}\quad \text{PERFOR}^s_t = 1/r_t\ (s_{t+h}/s_t)^{12/h}$$

$$\text{(C37)}\quad \text{PERFOR}^o_t = 1/r_t\ (p^o_{t+k}/p^o_t)^{12/k}$$

$$\text{(C38)}\quad \text{PERFOR}^m_t = 1/r_t\ (p^m_{t+k}/p^m_t)^{12/k}$$

$$\text{(C39)}\quad \text{NAIVE}^c_t = 1/r_t\ [(p^o_t/\Phi^o + p^m_t/\Phi^m)/s_t]^{12/k}$$

(C40) $NAIVE_t^s = 1/r_t\,(s_t/s_t)^{12/h} = 1/r_t$

(C41) $NAIVE_t^o = 1/r_t\,(p_t^o/p_t^o)^{12/k} = 1/r_t$

(C42) $NAIVE_t^m = 1/r_t\,(p_t^m/p_t^m)^{12/k} = 1/r_t$

k is 2 for t = January, March, May, June, October, and November; it is 3 for t = February, April, September, and December; it is 4 for t = August; and it is 5 for t = July. h is 2 for t = January, March, May, June, September, and November; it is 3 for t = February, April, August, and October; and it is 4 for t = July.